

# CrimeStat IV

## Part II: Spatial Description

**Chapter 4:**  
**Centrographic Statistics**

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## Chapter 4:

# Centrographic Statistics

In this chapter, the spatial distribution of crime incidents will be discussed. The statistics that are used in describing the spatial distribution of crime incidents will be explained and will be illustrated with examples from *CrimeStat*<sup>® III</sup>. For the examples, crime incident data from Baltimore County and Baltimore City will be used. Figure 4.1 shows the user interface for the spatial distribution statistics in *CrimeStat*. For each of these, the statistics will first be presented followed by examples of their use in crime analysis.

## Centrographic Statistics

The most basic type of descriptors for the spatial distribution of crime incidents are *centrographic statistics*. These are indices which estimate basic parameters about the distribution (Lefever, 1926; Furfey, 1927; Bachi, 1957; Neft, 1962, Hultquist, Brown and Holmes, 1971; Ebdon, 1988). They include:

1. Mean center
2. Median center
3. Center of minimum distance
4. Standard deviation of X and Y coordinates
5. Standard distance deviation
6. Standard deviational ellipse

They are called centrographic in that they are two dimensional correlates to the basic statistical moments of a single-variable distribution - mean, standard deviation, skewness, and kurtosis (see Bachi, 1957). They have been applied to crime analysis by Stephenson (1980) and by Langworthy and Jefferis (1998). Because two dimensions add complexity not seen in one dimension, these statistical moments have been modified to be appropriate. Figure 4.2 shows how the centrographic statistics are selected in *CrimeStat*.

## Mean Center

The simplest descriptor of a distribution is the *mean center*. This is merely the mean of the X and Y coordinates. It is sometimes called a *center of gravity* in that it represents the point in a distribution where all other points are balanced if they existed on a plane and the mean center was a fulcrum (Ebdon, 1988; Burt and Barber, 1996).

Figure 4.1:  
**Spatial Distribution Screen**

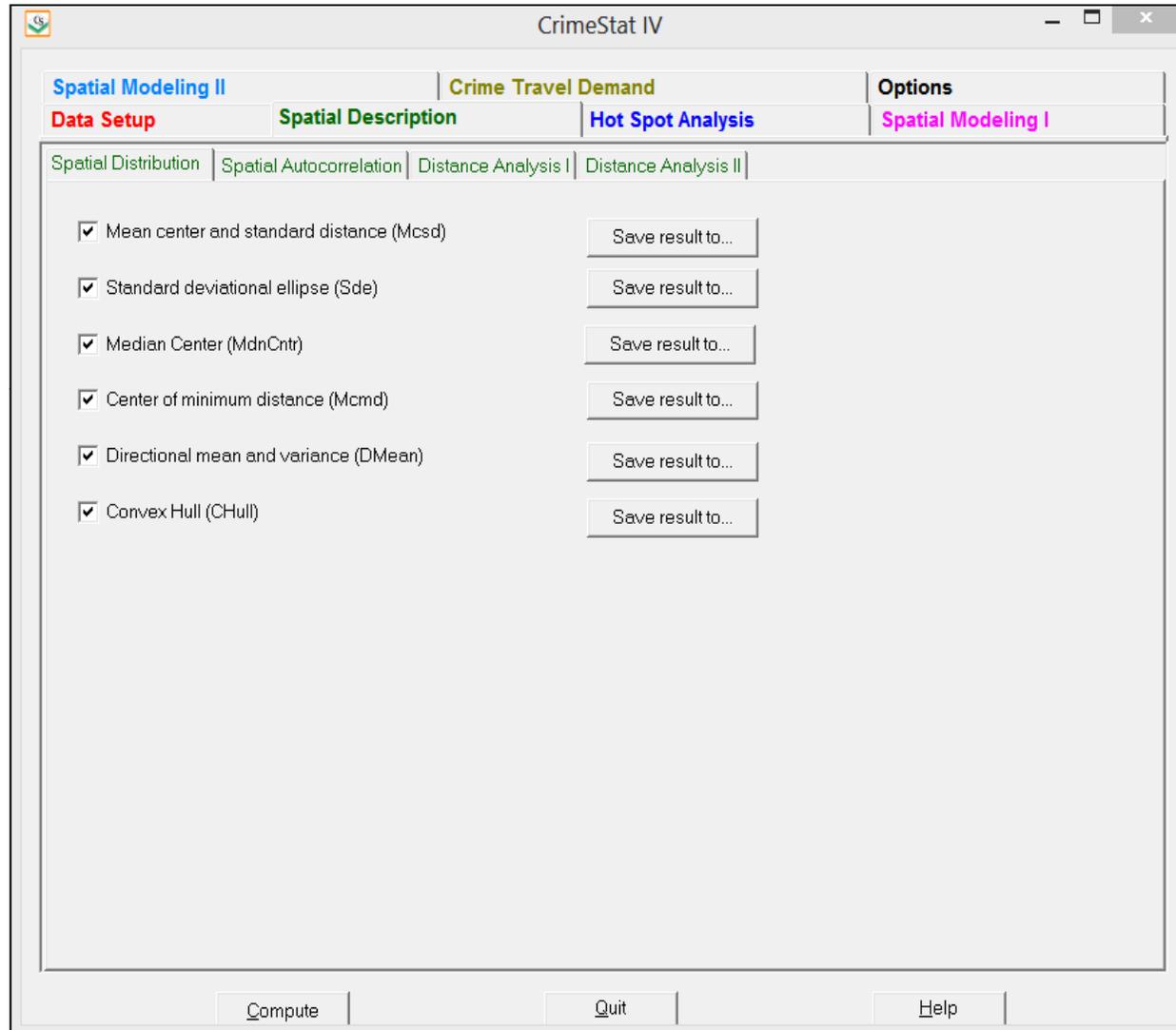
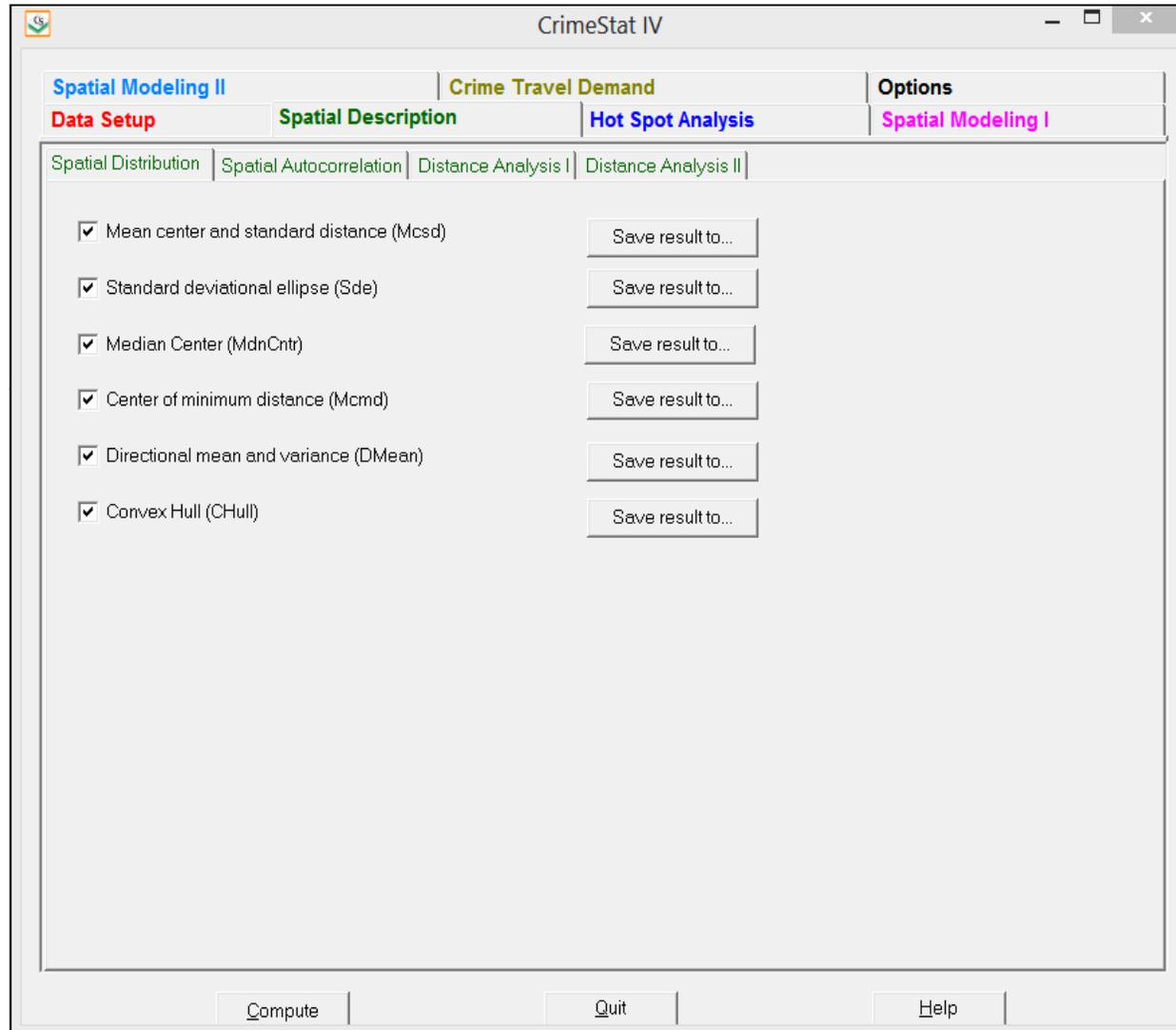


Figure 4.2:  
**Selecting Centrographic Statistics**



For a single variable, the mean is the point at which the sum of all differences between the mean and all other points is zero. Unfortunately, for two variables, such as the location of crime incidents, the mean center is not necessarily the point at which the sum of all distances to all other points is minimized. That property is attributed to the center of minimum distance (see below). However, the mean center can be thought of as a point where both the sum of all differences between the mean X coordinate and all other X coordinates is zero and the sum of all differences between the mean Y coordinate and all other Y coordinates is zero.

The formula for the mean center is:

$$\bar{X} = \frac{\sum_{i=1}^N X_i}{N} \quad (4.1)$$

$$\bar{Y} = \frac{\sum_{i=1}^N Y_i}{N} \quad (4.2)$$

where  $X_i$  and  $Y_i$  are the coordinates of individual locations and  $N$  is the total number of points. To take a simple example, the mean center for burglaries in Baltimore County has spherical coordinates of longitude -76.608482, latitude 39.348368 and for robberies longitude -76.620838, latitude 39.334816. Figure 4.3 illustrates these two mean centers.

## Weighted Mean Center

A *weighted mean center* is produced by weighting each coordinate by another variable,  $W_i$ . For example, if the coordinates are the centroids of census tracts, then the weight could be the population within the census tract. The weights have to be a positive number greater than or equal to 1. The numerator is the sum of the product of the variable and the weight while the denominator is the sum of weights,

$$\bar{X} = \frac{\sum_{i=1}^N W_i X_i}{\sum_{i=1}^N W_i} \quad (4.3)$$

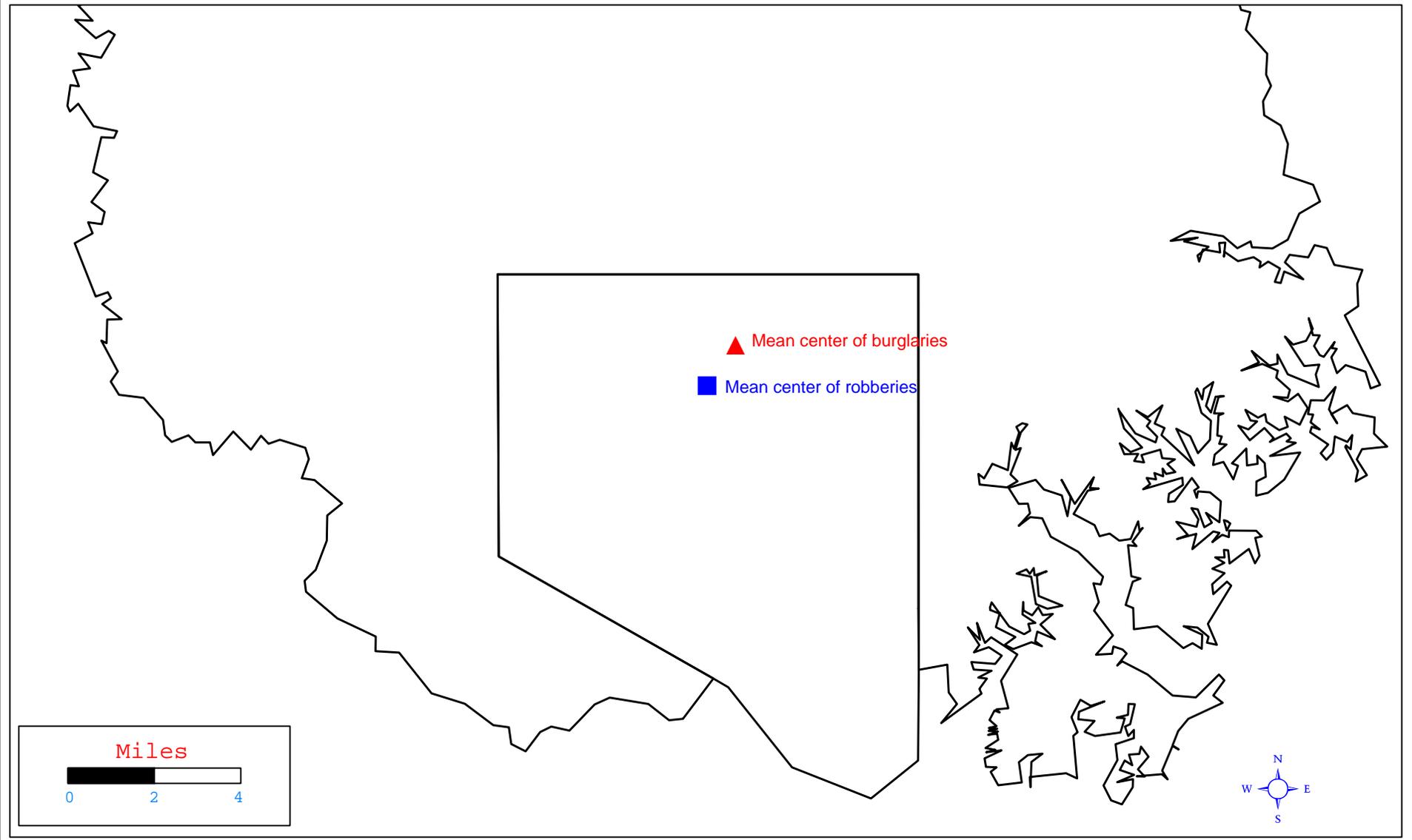
$$\bar{Y} = \frac{\sum_{i=1}^N W_i Y_i}{\sum_{i=1}^N W_i} \quad (4.4)$$

where  $W_i$  is the weight of observation  $i$  and  $X_i$  and  $Y_i$  are as defined in equations 4.1 and 4.2.

The advantage of a weighted mean center is that points associated with areas can have the characteristics of the areas included. For example, if the coordinates are the centroids of census

# Figure 4.3: Burglary and Robbery in Baltimore County

## Comparison of Mean Centers



tracts, then the weight of each centroid could be the population within the census tract. This will produce a different center of gravity than the unweighted center of all census tracts.

*CrimeStat* allows the mean to be weighted by either the weighting variable or by the intensity variable. Users should be careful, however, not to weight the mean with both the weighting and intensity variable unless there is an explicit distinction being made between weights and intensities.

To take an example, in the six jurisdictions making up the metropolitan Baltimore area (Baltimore City, and Baltimore, Carroll, Harford, Howard and Anne Arundel counties), the mean center of all census block groups is longitude -76.619121, latitude 39.304344. This would be an *unweighted* mean center of the block groups. On the other hand, the mean center of the 1990 population for the Baltimore metropolitan area had coordinates of longitude -76.625186 and latitude 39.304186, a position slightly southwest of the unweighted mean center. Weighting the block groups by median household income produces a mean center which is still more southwest. Figure 4.4 illustrates these three mean centers.

Weighted mean centers can be useful because they describe spatial differentiation in the metropolitan area and factors that may correlate with crime distributions. Another example is the weighted mean centers of different ethnic groups in the Baltimore metropolitan area (figure 4.5). The mean center of the White population is almost identical to the unweighted mean center. On the other hand, the mean center of the African-American/Black population is southwest of this and the mean center of the Hispanic/Latino population is considerably south of that for the White population. In other words, different ethnic groups tend to live in different parts of the Baltimore metropolitan area. Whether this has any impact on crime distributions is an empirical question.

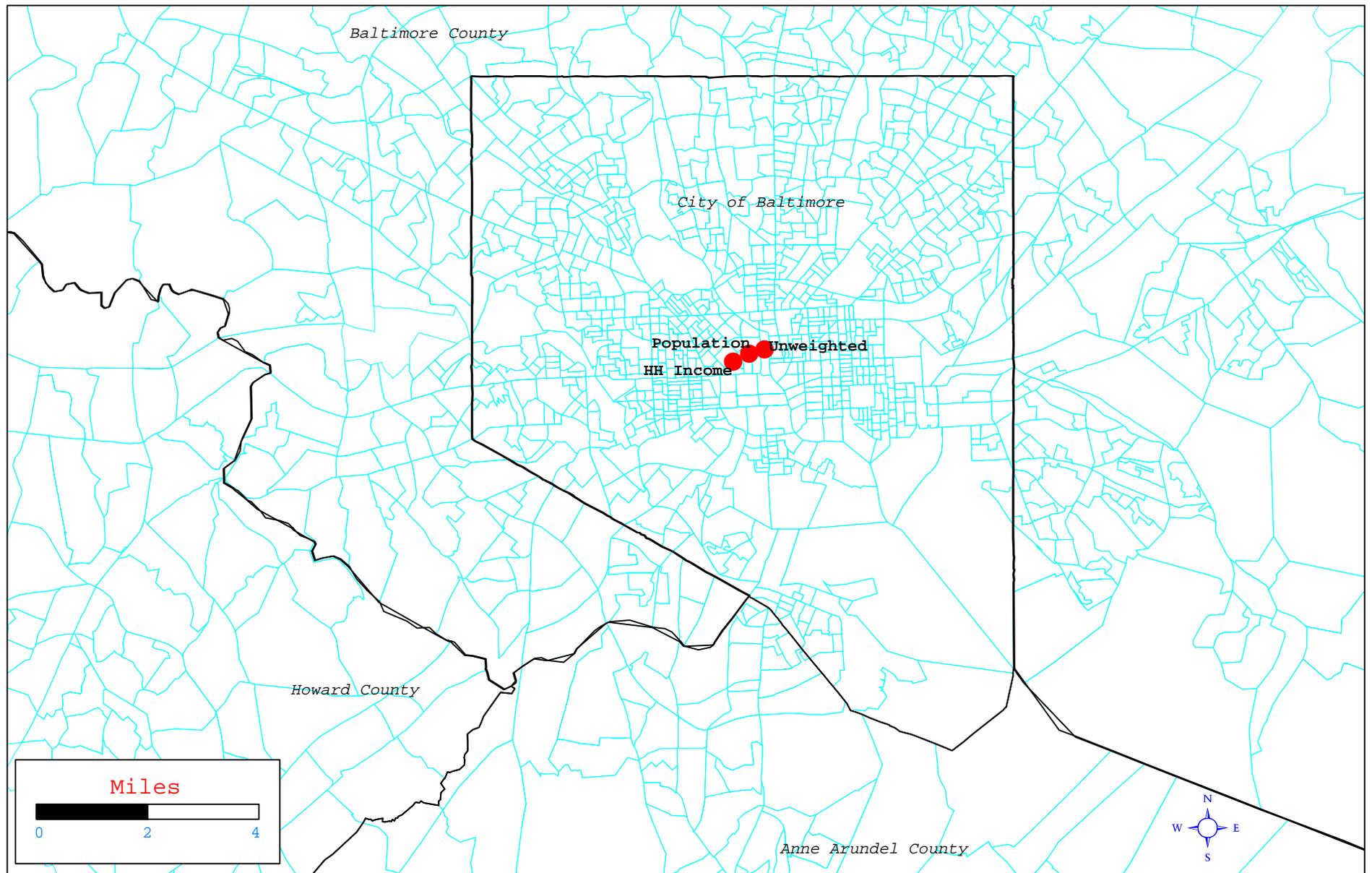
When the *Mcsd* box is checked, *CrimeStat* will run the routine. *CrimeStat* has a status bar that indicates how much of the routine has been run (Figure 4.6).<sup>1</sup> The results of these statistics are shown in the *Mcsd* output table (figure 4.7).

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1 Hint. There are 40 bars indicated in the status bar while a routine is running. For long runs, users can estimate the calculation time by timing how long it takes for two bars to be displayed and then multiply by 20.

# Figure 4.4: Center of Baltimore Metropolitan Population

Mean Center of Block Groups Weighted By Selected Variables



# Figure 4.5: Center of Baltimore Metropolitan Population

Mean Center of Block Groups Weighted By Selected Variables

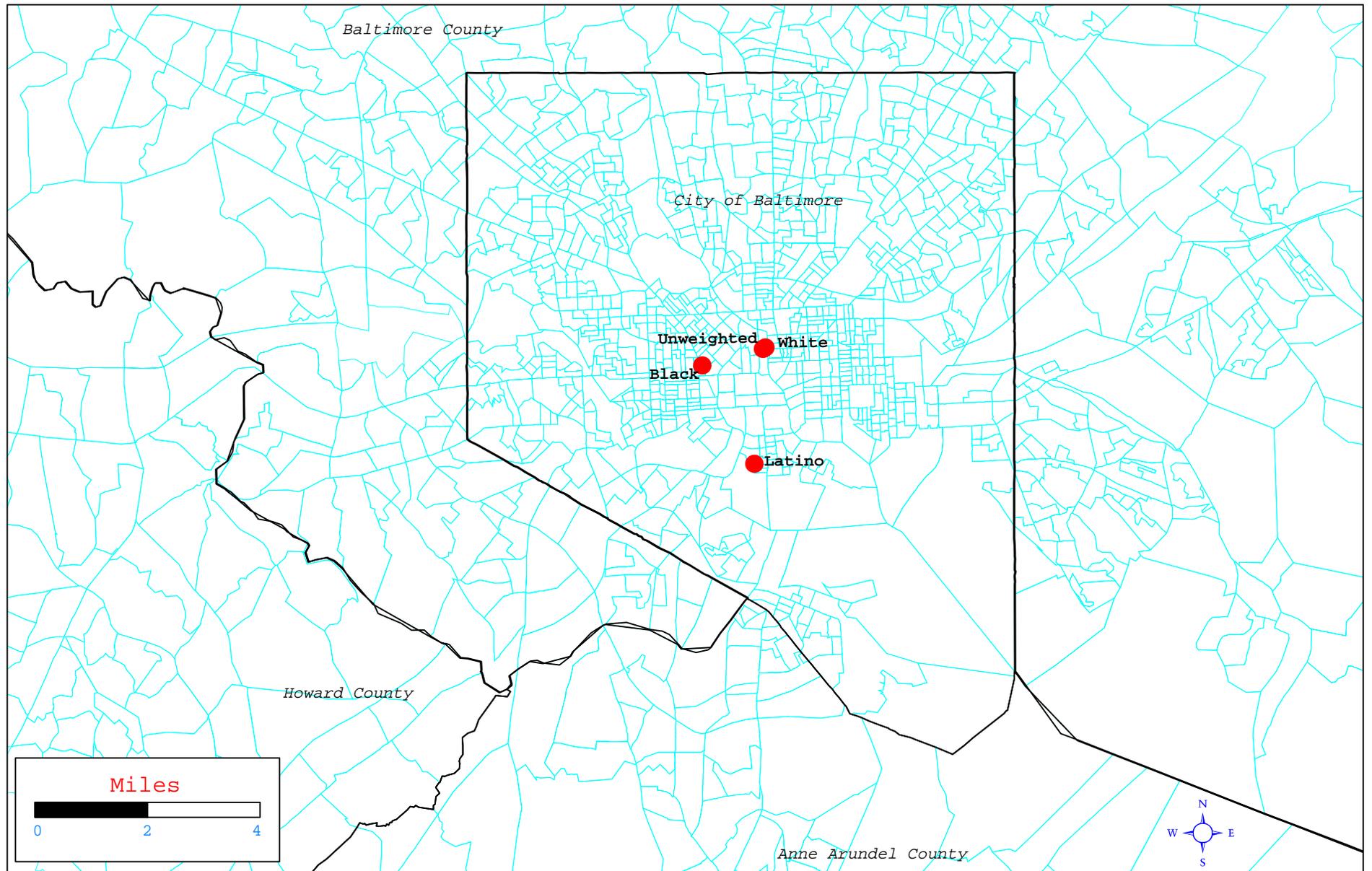


Figure 4.6:  
**CrimeStat Calculating a Routine**

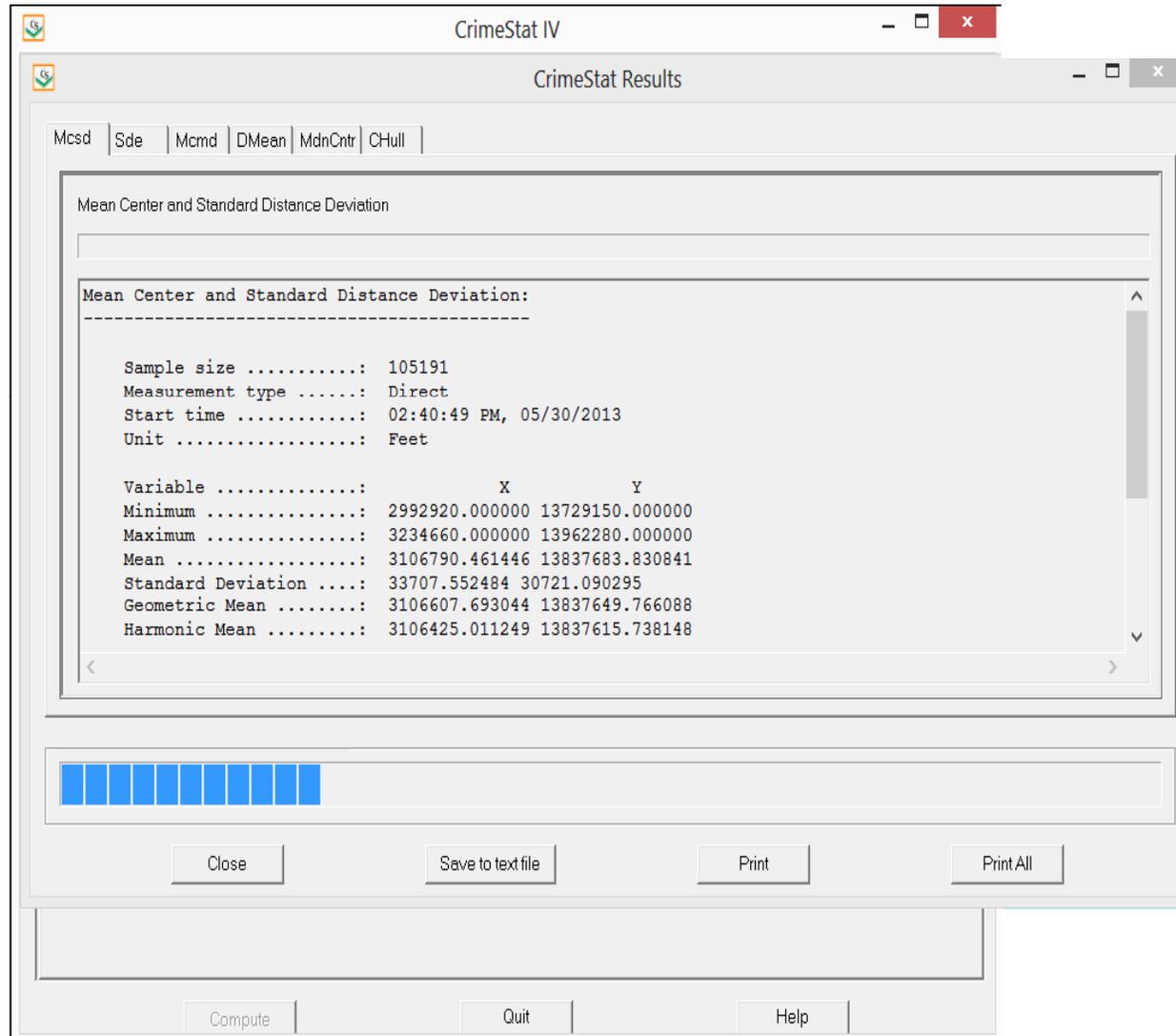
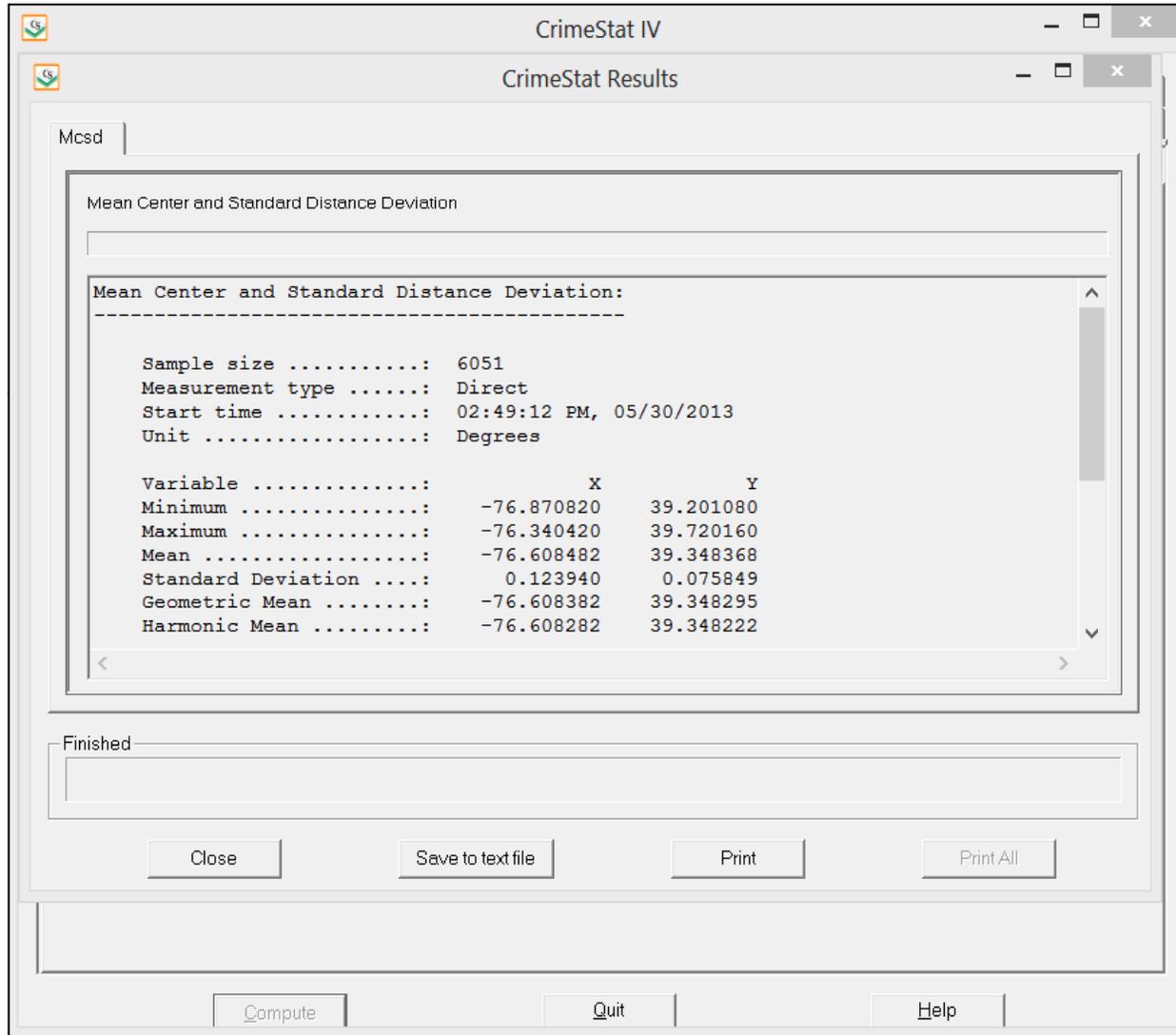
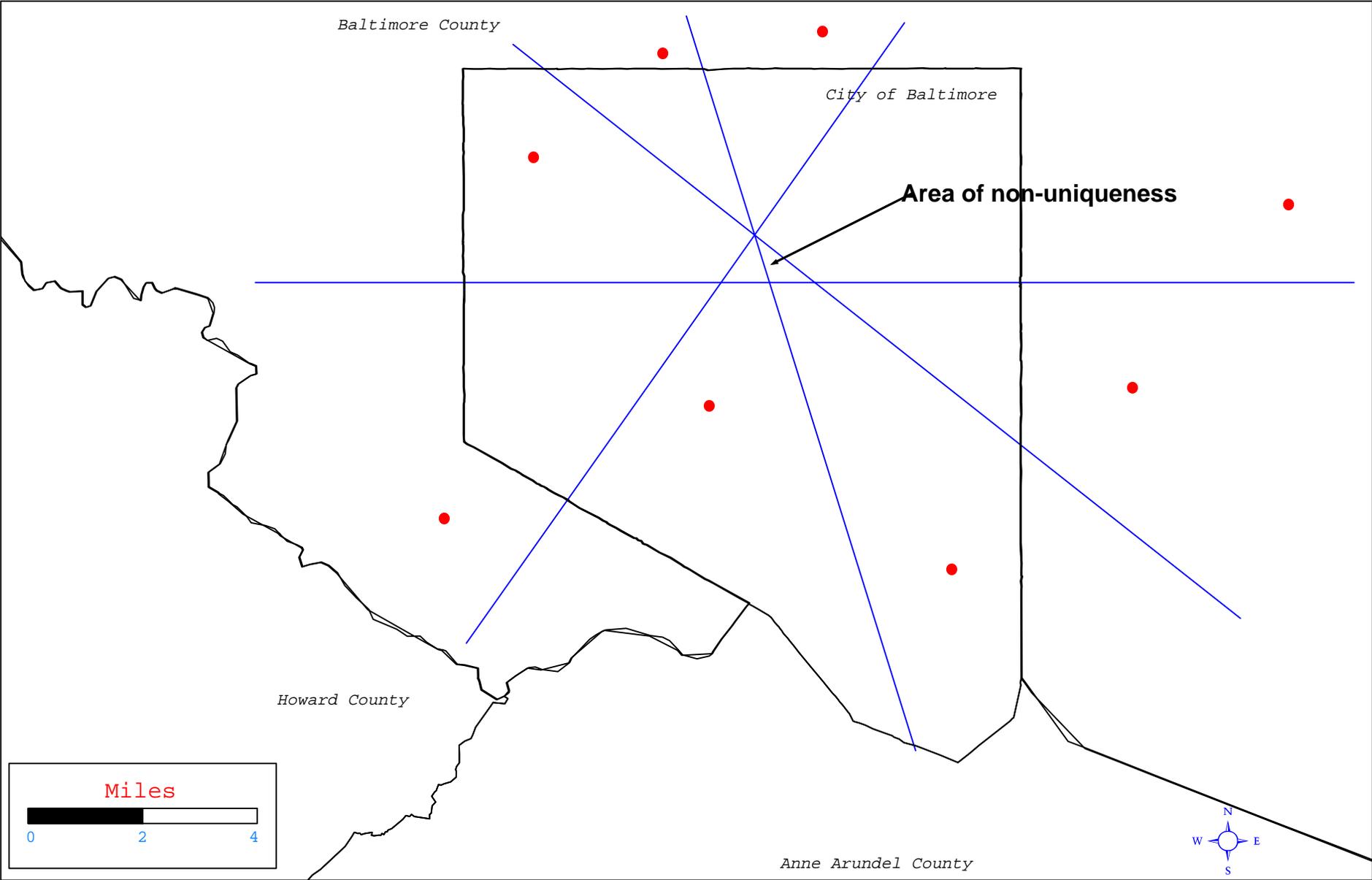


Figure 4.7:  
Mean Center and Standard Distance Deviation Output



# Figure 4.8: Non-Uniqueness of a Median Center

Lines Splitting Incident Locations Into Two Halves



## Median Center

The **median center** is the intersection between the median of the X coordinate and the median of the Y coordinate. The concept is simple. However, it is not strictly a median. For a single variable, such as median household income, the median is that point at which 50% of the cases fall below and 50% fall above. On a two dimensional plane, however, there is not a single median because the location of a median is defined by the way that the axes are drawn.

For example, in figure 4.8, there are eight incident points shown. Four lines have been drawn which divide these eight points into two groups of four each. However, the four lines do not identify an exact location for a median. Instead, there is an area of non-uniqueness in which any part of it could be considered the 'median center'. This violates one of the basic properties of a statistic is that it be a unique value.

Nevertheless, as long as the axes are not rotated, the median center can be a useful statistic. The *CrimeStat* routine outputs three statistics:

1. The sample size
2. The median of X
3. The median of Y

The tabular output can be printed and the median center can be output as a graphical object to *ArcGIS* 'shp', *MapInfo* 'mif', *Google Earth* 'kml, or various Ascii files. A root name should be provided. The median center is output as a point (MdnCntr<root name>).

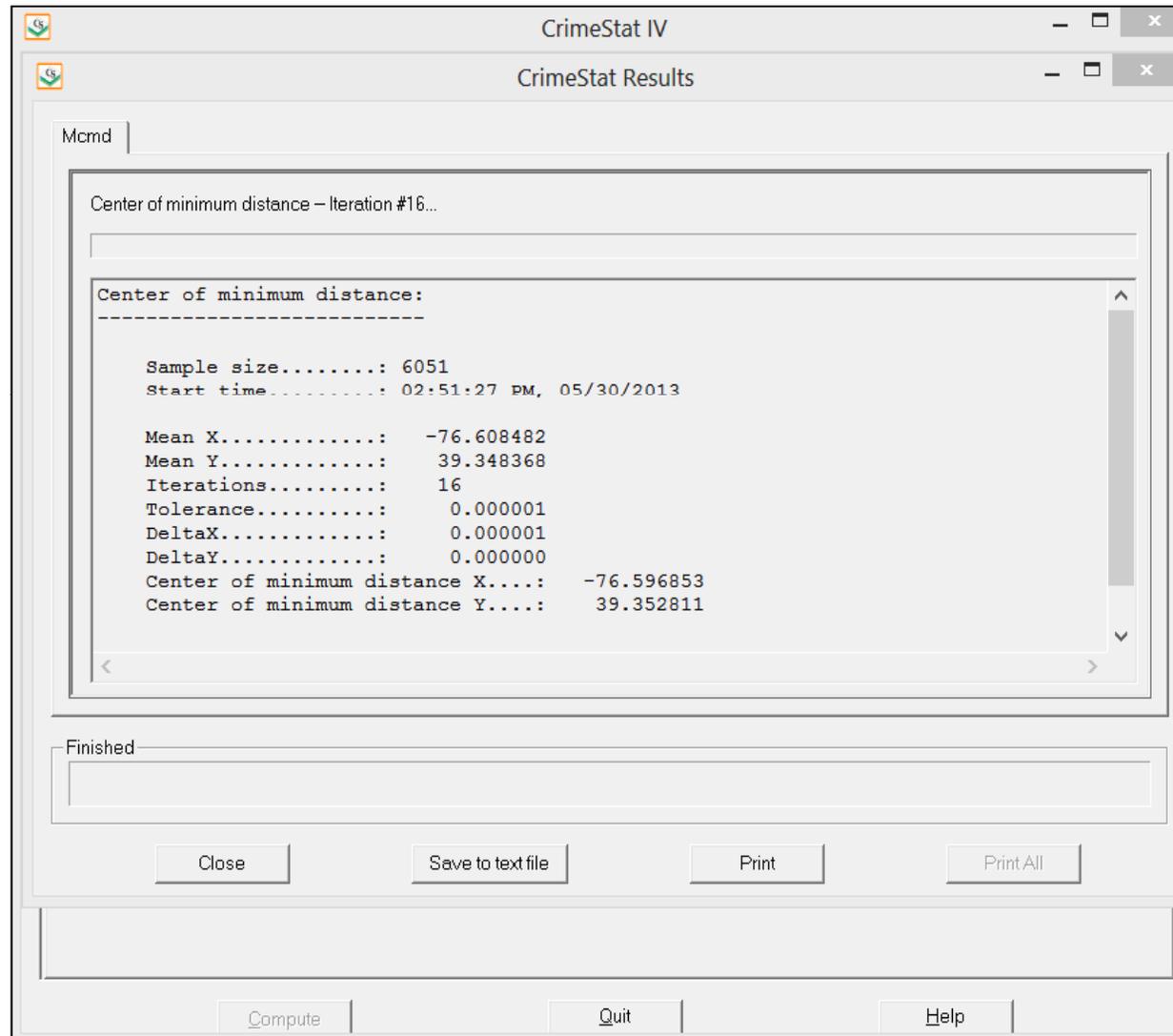
## Center of Minimum Distance

Another centrographic statistic is the **center of minimum distance**. Unfortunately, this statistic is sometimes also called the *median center*, which can make it confusing since the above statistic has the same name. Nevertheless, unlike the median center above, the center of minimum distance is a unique statistic in that it defines the point at which the sum of the distance to all other points is the smallest (Burt and Barber, 1996). It is defined as:

$$\text{Center of minimum distance} = CMD = \sum_{i=1}^N d_{ic} = \min \quad (4.5)$$

where  $d_{ic}$  is the distance between a single point,  $i$ , and  $C$ , the center of minimum distance (with an X and Y coordinate). Unfortunately, there is not a formula that can calculate this location.

Figure 4.9:  
**Center of Minimum Distance Output**



Instead, an iterative algorithm is used that approximates this location (Kuhn and Kuenne, 1962; Burt and Barber, 1996; see endnote *i*). Depending on whether the coordinates are spherical or projected, *CrimeStat* will calculate distance as either Great Circle (spherical) or Euclidean (projected), as discussed in the previous chapter. The results are shown in the *Mcmd* output table (figure 4.9).

The importance of the center of minimum distance is that it is a location where distance to all the defining incidents is the smallest. Since *CrimeStat* only measures distances as either direct or indirect, actual travel time is not being calculated. But in many jurisdictions, the minimum distance to all points is a good approximation to the point where travel distances are minimized. For example, in a police precinct, a patrol car could be stationed at the center of minimum distance to allow it to respond quickly to calls for service.

For example, figure 4.10 maps the center of minimum distance for 1996 auto thefts in both Baltimore City and Baltimore County and compares this to both the mean center and the median center statistic. As seen, both the center of minimum distance and the median center are south of the mean center, indicating that there are slightly more incidents in the southern part of the metropolitan area than in the northern part. However, the difference in these three statistics is very small, especially the median center and the center of minimum distance.

## Standard Deviations of the X and Y Coordinates

In addition to the mean center and center of minimum distance, *CrimeStat* will calculate various measures of spatial distribution, which describe the dispersion, orientation, and shape of the distribution of a variable (Hammond & McCulloch 1978; Ebdon 1988). The simplest of these is the raw **standard deviations of the X and Y coordinates**, respectively. The formulas used are the standard ones found in most elementary statistics books:

$$s_x = \sqrt{\sum_{i=1}^N \frac{(X_i - \bar{X})^2}{N-1}} \quad (4.6)$$

$$s_y = \sqrt{\sum_{i=1}^N \frac{(Y_i - \bar{Y})^2}{N-1}} \quad (4.7)$$

where  $X_i$  and  $Y_i$  are the X and Y coordinates for individual points,  $\bar{X}$  and  $\bar{Y}$  are the means of X and Y respectively, and N is the total number of points. Note that 1 is subtracted from the number of points to produce an unbiased estimate of the standard deviation.

# Figure 4.10: 1996 Metropolitan Baltimore Auto Thefts

Mean Center and Center of Minimum Distance for 1996 Auto Thefts

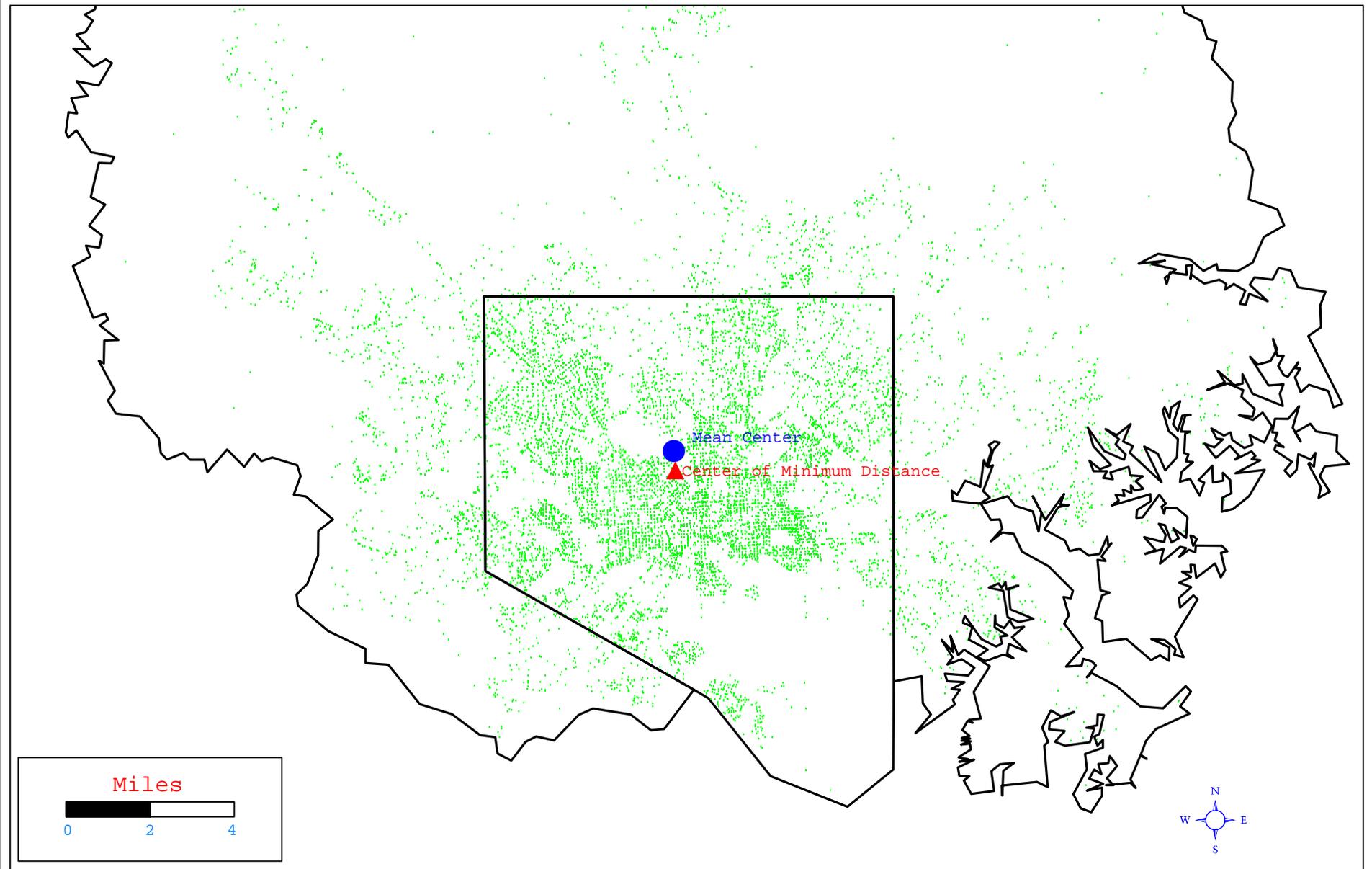


Figure 4.11 shows the standard deviation of the coordinates for auto thefts and represents this as a rectangle. As seen, the distribution of auto thefts spreads more in an east-west direction than in a north-south direction.

## Standard Distance Deviation

While the standard deviation of the X and Y coordinates provides some information about the dispersion of the incidents, there are two problems with it. First, it does not provide a single summary statistic of the dispersion in the locations and is actually two separate statistics, the dispersion in X and the dispersion in Y. Second, it provides measurement in the units of the coordinate system. Thus, if spherical coordinates are being used, then the units will be decimal degrees. On the other hand, if projected coordinates are being used, then units will be in feet or meters or some other metric.

A measure which overcomes these problems is the **standard distance deviation** (or *standard distance*, for short). This is the standard deviation of the *distance* of each point from the mean center and is expressed in measurement units (feet, meters, miles). It is the two-dimensional equivalent of a standard deviation.

The formula for it is:

$$SDD = \sqrt{\sum_{i=1}^N \frac{(d_{iMC})^2}{N-2}} \quad (4.8)$$

where  $d_{iMC}$  is the distance between each point,  $i$ , and the mean center and  $N$  is the total number of points. Note that 2 is subtracted from the number of points to produce an unbiased estimate of standard distance since there are two constants from which this distance is measured (mean of X, mean of Y).<sup>2</sup>

The standard distance can be represented as a single vector rather than two vectors as with the standard deviation of the X and Y coordinates. Figure 4.12 shows the mean center and standard distance deviation of both robberies and burglaries for 1996 in Baltimore County

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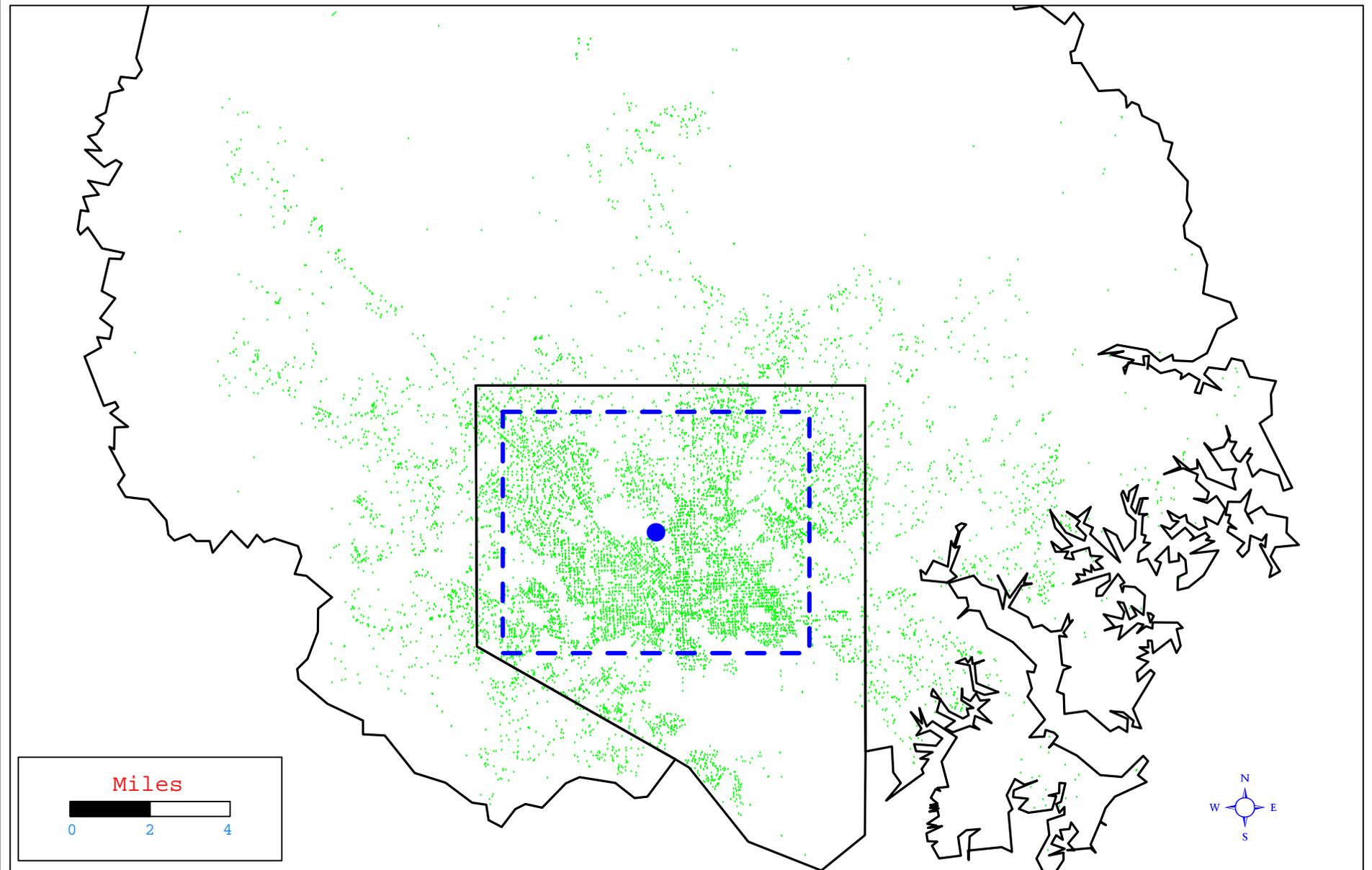
2 With a weight for an observation,  $w_i$ , the squared distance is weighted and the formula becomes:

$$s_{XY} = \sqrt{\frac{\sum w_i d_{iMC}^2}{(\sum w_i) - 2}}$$

where  $d_{iMC}$  is the distance from the point to the mean center. Both summations are over all points,  $N$ .

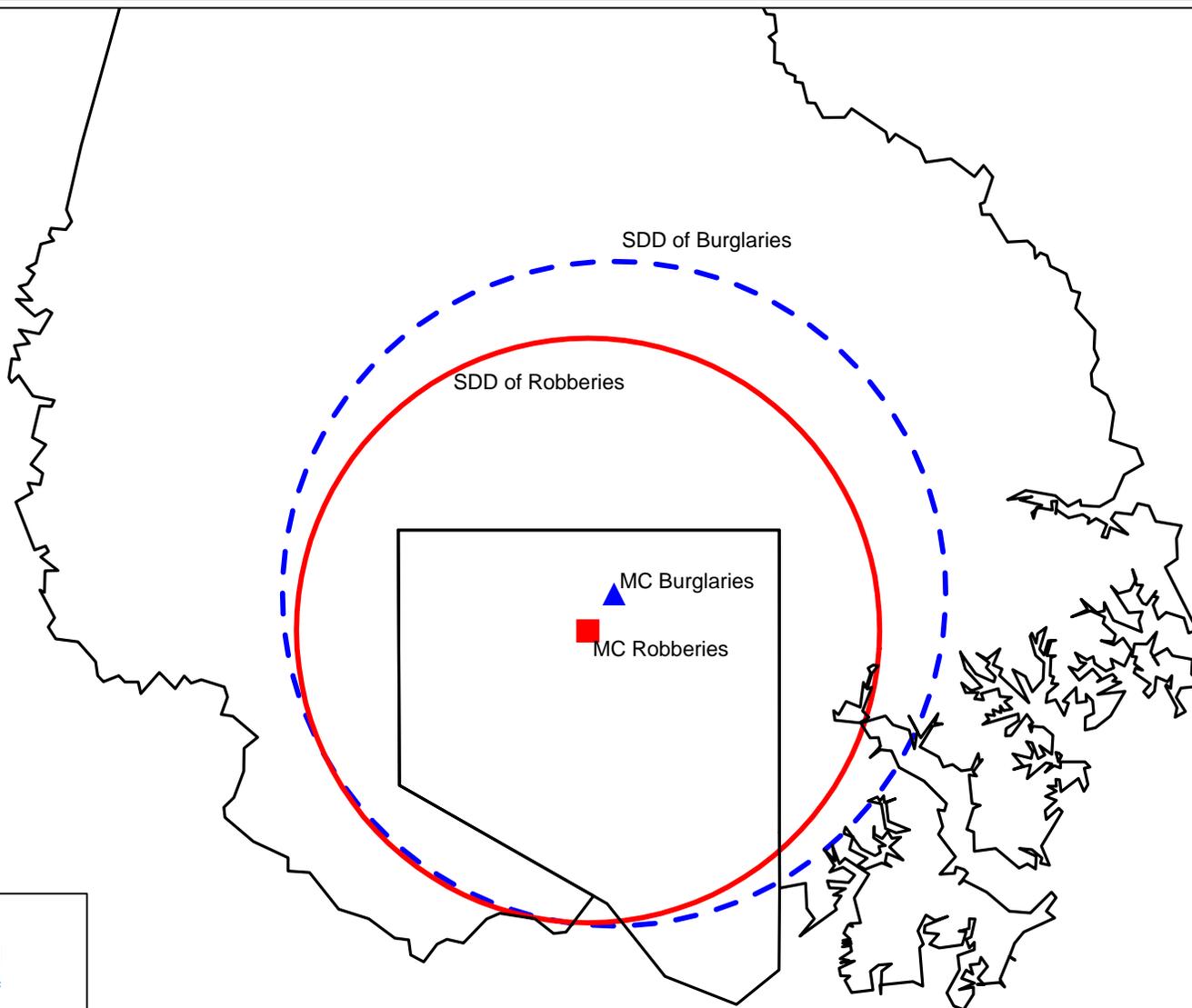
# Figure 4.11: 1996 Metropolitan Baltimore Auto Thefts

Mean Center and Standard Deviations of X and Y Coordinates



# Figure 4.12: 1996 Baltimore County Burglaries and Robberies

Comparison of Mean Centers and Standard Distance Deviations



represented as circles. It is clear that the spatial distributions of these two types of crime vary with robberies being slightly more concentrated.

## Standard Deviation Ellipse

The standard distance deviation is a good single measure of the dispersion of the incidents around the mean center. However, with two dimensions, distributions are frequently skewed in one direction or another (a condition called *anisotropy*).

Instead, there is another statistic that gives dispersion in two dimensions, the **standard deviational ellipse** (or *ellipse*, for short; Ebdon, 1988; Cromley, 1992). The standard deviational ellipse is derived from the bivariate distribution (Furfey, 1927; Neft, 1962; Bachhi, 1957) and is defined by:

$$\text{Bivariate distribution} = \sqrt{\frac{(s_x^2 + s_y^2)}{2}} \quad (4.9)$$

The two standard deviations, in the X and Y directions, are orthogonal to each other and define an ellipse. Ebdon (1988) rotates the X and Y axis so that the sum of squares of distances between points and axes are minimized. By convention, it is shown as an ellipse.

Aside from the mean X and mean Y, the formulas for these statistics are as follows (the observation subscript, *i*, has been dropped from the summation sign):

1. The Y-axis is rotated *clockwise* through an angle,  $\theta$ , where

$$\theta = \arctan \left\{ \frac{[\Sigma(x_i - \bar{x})^2 - \Sigma(y_i - \bar{y})^2] + \sqrt{[\Sigma(x_i - \bar{x})^2 - \Sigma(y_i - \bar{y})^2]^2 + 4[\Sigma(x_i - \bar{x})(y_i - \bar{y})]^2}}{2\Sigma(x_i - \bar{x})(y_i - \bar{y})} \right\} \quad (4.10)$$

where all summations are for  $i=1$  to  $N$  (Ebdon, 1988).

2. Two standard deviations are calculated, one along the transposed X-axis and one along the transposed Y-axis:

$$s_x = \sqrt{\frac{\Sigma[(x_i - \bar{x})\cos\theta - (y_i - \bar{y})\sin\theta]^2}{N-2}} \quad (4.11)$$

$$s_Y = \sqrt{\frac{\sum[(x_i - \bar{X})\sin\theta + (Y_i - \bar{Y})\cos\theta]^2}{N-2}} \quad (4.12)$$

where  $\bar{X}$  and  $\bar{Y}$  are the means of X and Y respectively,  $\theta$  is the angle (in radians), and N is the number of points. Note, again, that 2 is subtracted from the number of points in both denominators to produce an unbiased estimate of the standard deviational ellipse since there are two constants from which the distance along each axis is measured ( $\bar{X}$ ,  $\bar{Y}$ ; see endnote *ii*).

3. The X-axis and Y-axis of the ellipse are defined by:

$$Length_X = 2s_X \quad (4.13)$$

$$Length_Y = 2s_Y \quad (4.14)$$

4. The area of the ellipse is:

$$A = \pi s_X s_Y \quad (4.15)$$

Figure 4.13 shows the output of the ellipse routine and figure 4.14 maps the standard deviational ellipse of auto thefts in Baltimore City and Baltimore County for 1996.

## Geometric Mean

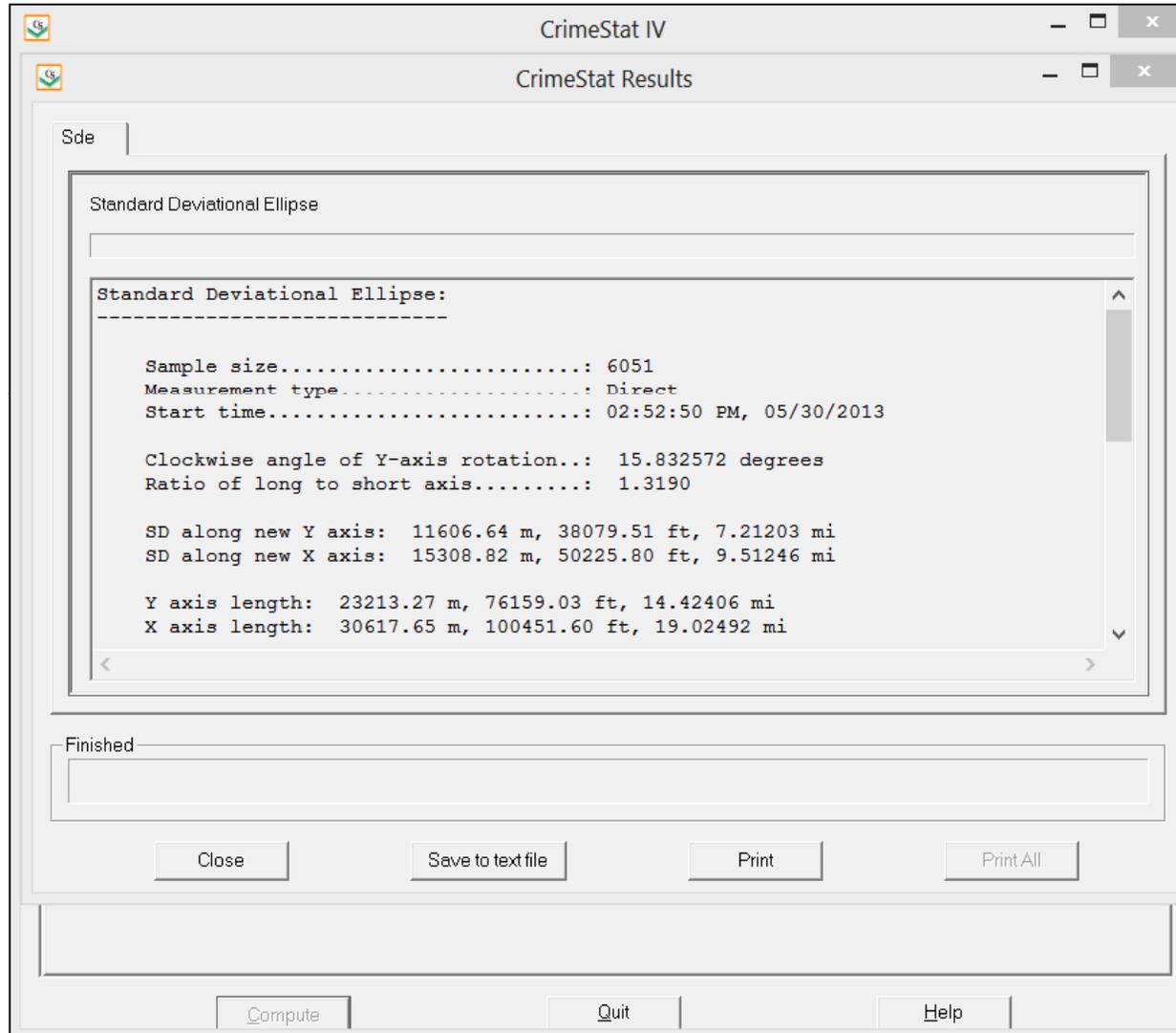
The mean center routine (Mcsd) includes two additional means. First, there is the **geometric mean**, which is a mean associated with the mean of the logarithms. It is defined as:

$$Geometric\ Mean\ of\ X = GM(X) = \prod_{j=1}^N (X_i^{W_i})^{\frac{1}{\sum W_i}} \quad (4.16)$$

$$Geometric\ Mean\ of\ Y = GM(Y) = \prod_{j=1}^N (Y_i^{W_i})^{\frac{1}{\sum W_i}} \quad (4.17)$$

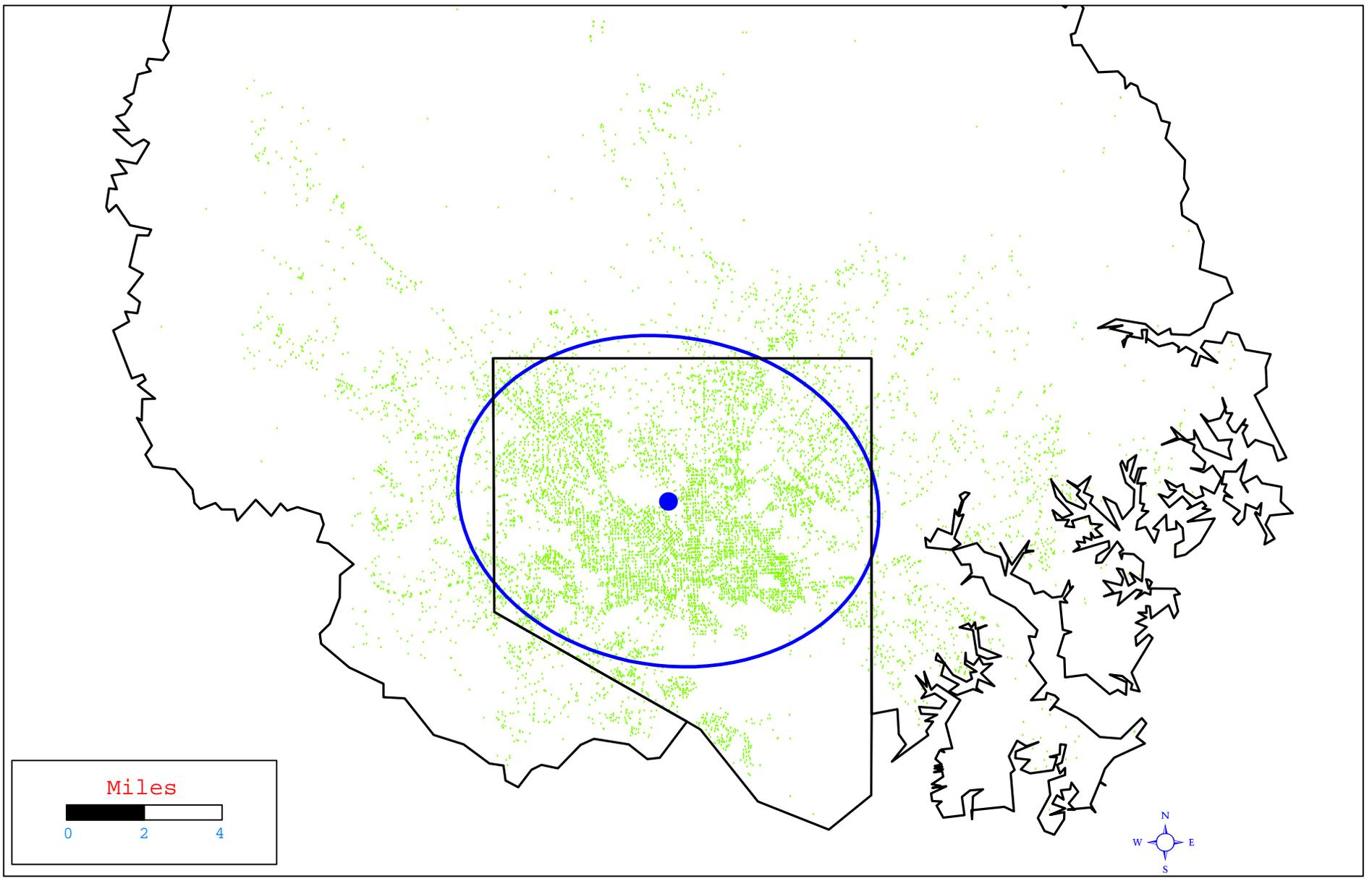
where  $\Pi$  is the product term of each point value,  $i$  (i.e., the values of X or Y are multiplied times each other),  $W_i$  is the weight used (default=1), and N is the sample size (Everitt, 2011). The weights must be defined on the Primary File page, either in the Weights field or in the Intensity field (but not both together).

Figure 4.13:  
**Standard Deviational Ellipse Output**



# Figure 4.14: 1996 Metropolitan Baltimore Auto Thefts

Mean Center and Standard Deviational Ellipse



The equation can be evaluated by logarithms:

$$\begin{aligned} \text{Ln}[GM(X)] &= \frac{1}{\sum W_i} [W_1 \text{Ln}(X_1) + W_2 \text{Ln}(X_2) + \dots + W_N \text{Ln}(X_N)] \\ &= \frac{\sum [W_i \text{Ln}(X_i)]}{\sum W_i} \end{aligned} \quad (4.18)$$

$$\text{Ln}[GM(Y)] = \frac{1}{\sum W_i} [W_1 \text{Ln}(Y_1) + W_2 \text{Ln}(Y_2) + \dots + W_N \text{Ln}(Y_N)] = \frac{\sum [W_i \text{Ln}(Y_i)]}{\sum W_i} \quad (4.19)$$

$$GM(X) = e^{\text{Ln}[GM(X)]} \quad (4.20)$$

$$GM(Y) = e^{\text{Ln}[GM(Y)]} \quad (4.21)$$

The geometric mean is the anti-log of the mean of the logarithms. If weights are used, then the logarithm of each X or Y value is weighted and the sum of the weighted logarithms are divided by the sum of the weights. If weights are not used, then the default weight is 1 and the sum of the weights will equal the sample size. The geometric mean is output as part of the Mcsd routine and has a 'Gm' prefix before the user defined name.

### Uses

The geometric mean is used when units are multiplied by each other (e.g., robberies increase by 5% one year, 3% the next, and 4% the next; Wikipedia, 2007a). One cannot take the simple mean because there is a cumulative change in the units. In most cases, this is not relevant to point (incident) locations since the coordinates of each incident are independent and are not multiplied by each other. However, the geometric mean can be useful because it converts all X and Y coordinates into logarithms and, thus, has the effect of discounting extreme values.

### Harmonic Mean

The **harmonic mean** is also a mean which discounts extreme values, but is calculated differently. It is defined as (Wikipedia, 2007b):

$$\text{Harmonic mean of } X = HM(X) = \frac{\sum W_i}{\sum (W_i/X_i)} \quad (4.22)$$

$$\text{Harmonic mean of } Y = HM(Y) = \frac{\sum W_i}{\sum (W_i/Y_i)} \quad (4.23)$$

where  $W_i$  is the weight used (default=1),  $X_i$  and  $Y_i$  are the X and Y values, and N is the sample size. The weights have to be defined on the Primary File page, either in the Weights field or in the Intensity field (but not both together).

The harmonic mean of X and Y is the inverse of the mean of the inverse of X and Y respectively (i.e., take the inverse; take the mean of the inverse; and invert the mean of the inverse). If weights are used, then each X or Y value is weighted by its inverse while the numerator is the sum of the weights. If weights are not used, then the default weight is 1 and the sum of weights will equal the sample size. The harmonic mean is output as part of the Mcsd routine and has a 'Hm' prefix before the user-defined name.

### Uses

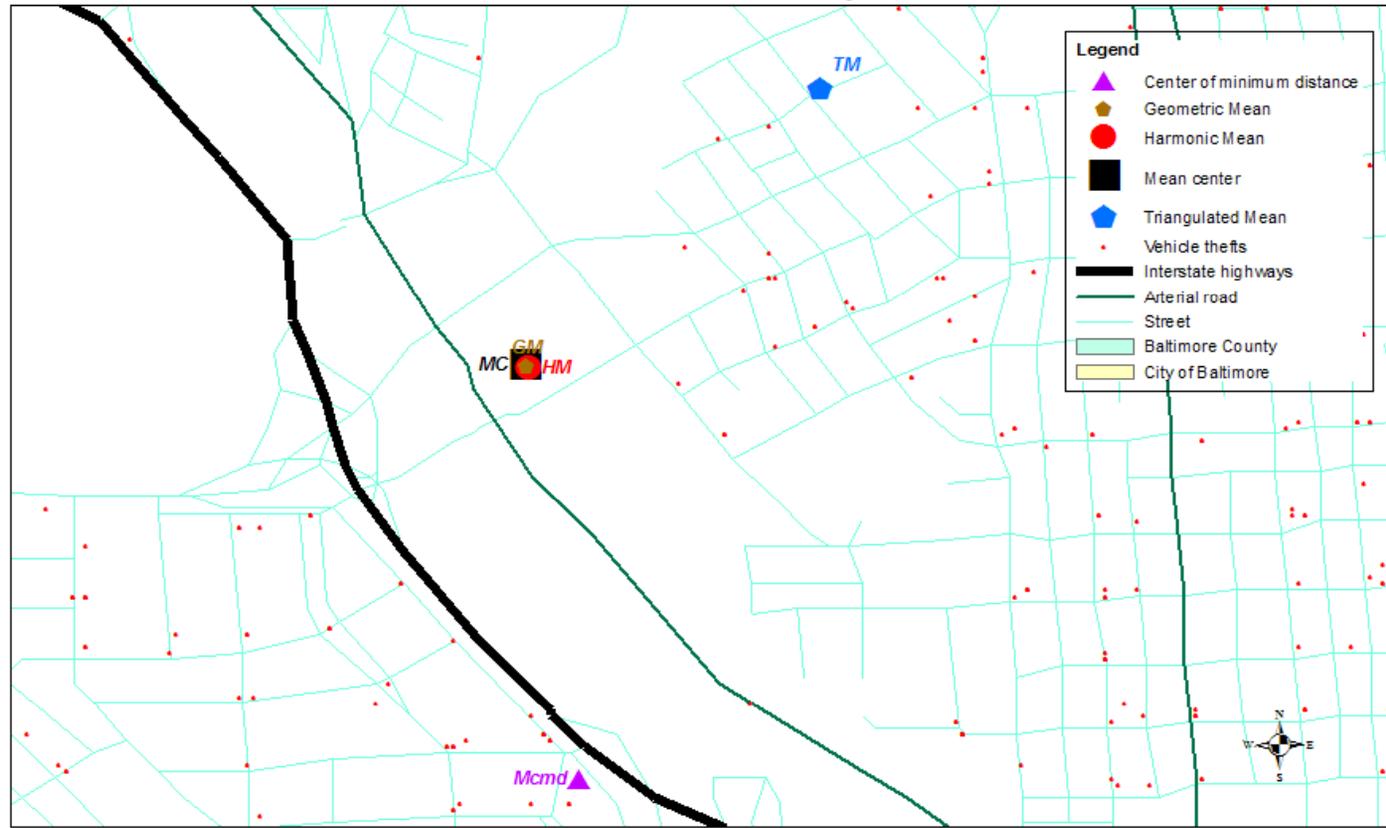
Typically, harmonic means are used in calculating the average of rates, or quantities whose values are changing over time (Wikipedia, 2007b). For example, in calculating the average speed over multiple segments of equal length (see chapter 30 on Network Assignment), the harmonic mean should be used, not the arithmetic mean. If there are two adjacent road segments, each one mile in length and if a car travels over the first segment 20 miles per hour (mph) but over the second segment at 40 mph, the average speed is not 30 mph (the arithmetic mean), but 26.7 mph (the harmonic mean). The car takes 3 minutes to travel the first segment (60 minutes per hour times 1 mile divided by 20 mph ) and 1.5 minutes to travel the second segment (60 minutes per hour times 1 mile divided by 40 mph). Thus, the total time to travel the two miles is 4.5 minutes and the average speed is 26.7 mph.

Again, for point (incident) locations, the harmonic mean would normally not be relevant since the coordinates of each of the incidents are independent. However, since the harmonic mean is weighted more heavily by the smaller values, it can be useful to discount cases which have outlying coordinates.

In other words, the harmonic mean of X and Y respectively is the inverse of the mean of the inverse of X and Y respectively (i.e., take the inverse; take the mean of the inverse; and invert the mean of the inverse). If weights are used, then each X or Y value is weighted and the numerator is the sum of the weights. If weights are not used, then the sum of the weights will equal the sample size. The harmonic mean is output as part of the Mcsd routine and has a 'Hm' prefix before the user defined name.

The geometric and harmonic means are discounted means that 'hug' the center of the distribution. They differ from the mean center when there is a very skewed distribution.

Figure 4.15:  
**1996 Baltimore Metropolitan Vehicle Thefts**  
 5 Mean Centers Compared



To contrast the different means, figure 4.15 below shows five different means for Baltimore County motor vehicle thefts:

1. Mean center;
2. Center of minimum distance;
3. Geometric mean;
4. Harmonic mean; and
5. Triangulated mean (discussed below)

In the example, the mean center, geometric mean, and harmonic mean fall very close to each other; however, they will not always be so. The center of minimum distance approximates the geographical center of the distribution. The triangulated mean is defined by the angularity and distance from the lower-left and upper-right corners of the data set (see below).

Centrographic descriptors can be very powerful tools for examining spatial patterns. They are a first step in any spatial analysis, but an important one. The above example illustrates how they can be a basis for decision-making, even with small samples. A couple of other examples can be illustrated.

## **Average Density**

The **average density** is the number of incidents divided by the area. It is a measure of the average number of events per unit of area; it is sometimes called the *intensity*. If the area is defined on the measurement parameters page, the routine uses that value for area; otherwise, it takes the rectangular area defined by the minimum and maximum X and Y values (the bounding rectangle).

## **Output Files**

### **Calculating the Statistics**

Once the statistics have been selected, the user clicks on *Compute* to run the routine. The results are shown in a results table.

## Tabular Output

For each of these statistics, *CrimeStat* produces tabular output. In *CrimeStat*, all tables are labeled by symbols, for example Mcds for the mean center and standard distance deviation or Mcmd for the center of minimum distance. All tables present the sample size.

## Graphical Objects

The six centographic statistics can be output as graphical objects. The mean center and center of minimum distance are output as single points. The standard deviation of the X and Y coordinates is output as a rectangle. The standard distance deviation is output as a circle and the standard deviational ellipse is output as an ellipse.

*CrimeStat* currently supports graphical outputs to *ArcGIS* 'shp', *MapInfo* 'mif', *Google Earth* 'kml', or various Ascii files. Before running the calculation, the user should select the desired output files and specify a root name (e.g., Precinct1Burglaries). Figure 4.16 shows a dialog box for outputting a shape file to *ArcGIS*. For *MapInfo* output only, the user has to also indicate the name of the projection, the projection number and the datum number. These can be found in the *MapInfo* users guide. By default, *CrimeStat* will use the standard parameters for a spherical coordinate system (Earth projection, projection number 1, and datum number 33). If a user requires a different coordinate system, the appropriate values should be typed into the space. Figure 4.17 shows the selection of the *MapInfo* coordinate parameters.

If requested, the output files are saved in the specified directory under the specified (root) name. For each statistic, *CrimeStat* will add prefix letters to the root name.

MC<root> for the mean center

MdnCntr<root> for the median center

Mcmd<root> for center of minimum distance

XYD<root> for the standard deviation of the X and Y coordinates

SDD<root> for the standard distance deviation

SDE<root> for the standard deviational ellipse.

The '.shp' files can be read directly into *ArcGIS* as themes. The 'kml' files can be read directly into Google Earth. The '.mif' files have to be imported into *MapInfo*.<sup>3</sup>

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3 In *MapInfo*, the command is Table Import <*MapInfo Interchange file*> though it is a lot easier to use the *MapInfo* Universal Translator.

Figure 4.16:  
**Outputting a Shape File to *ArcView/ArcGIS***

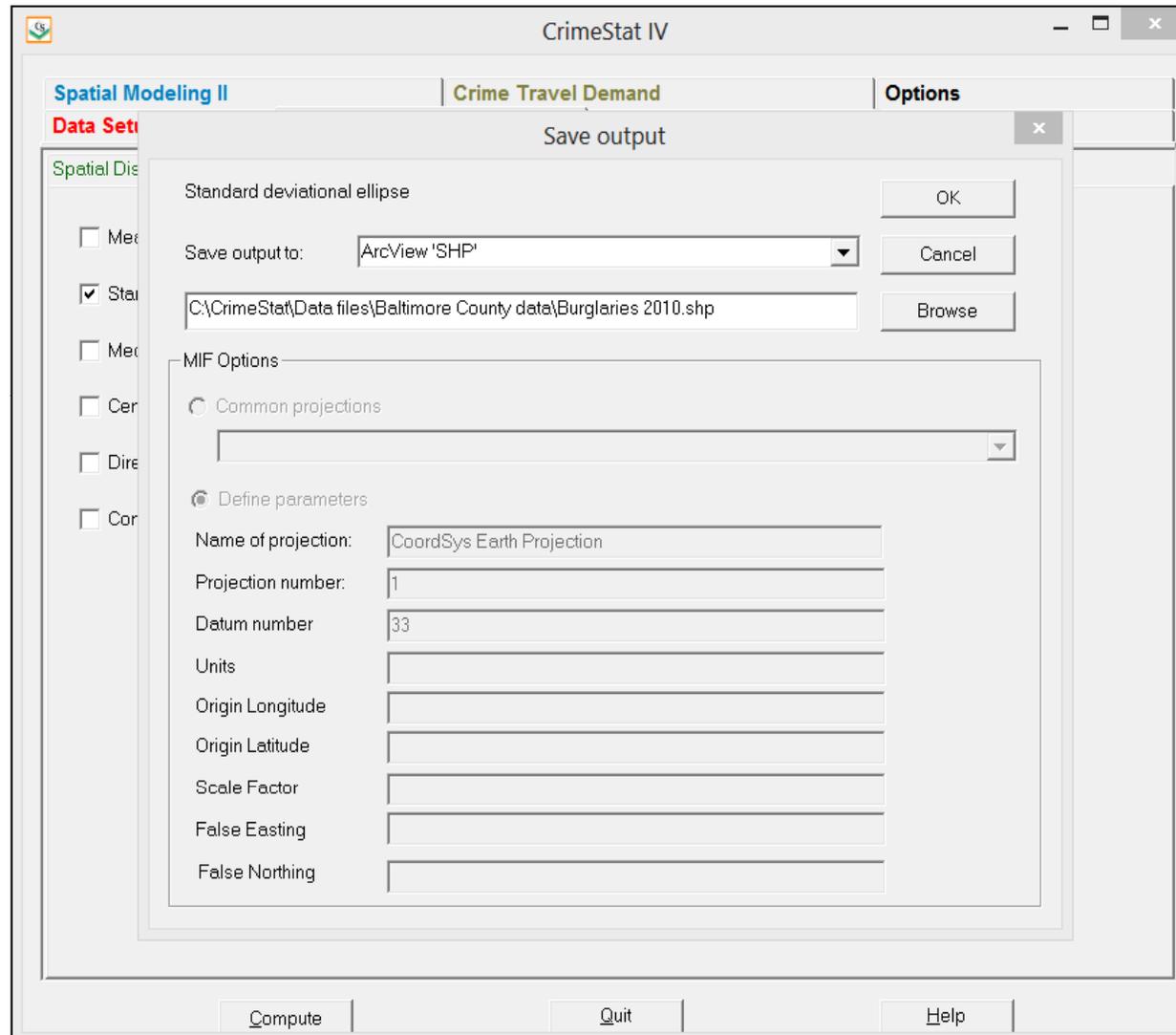
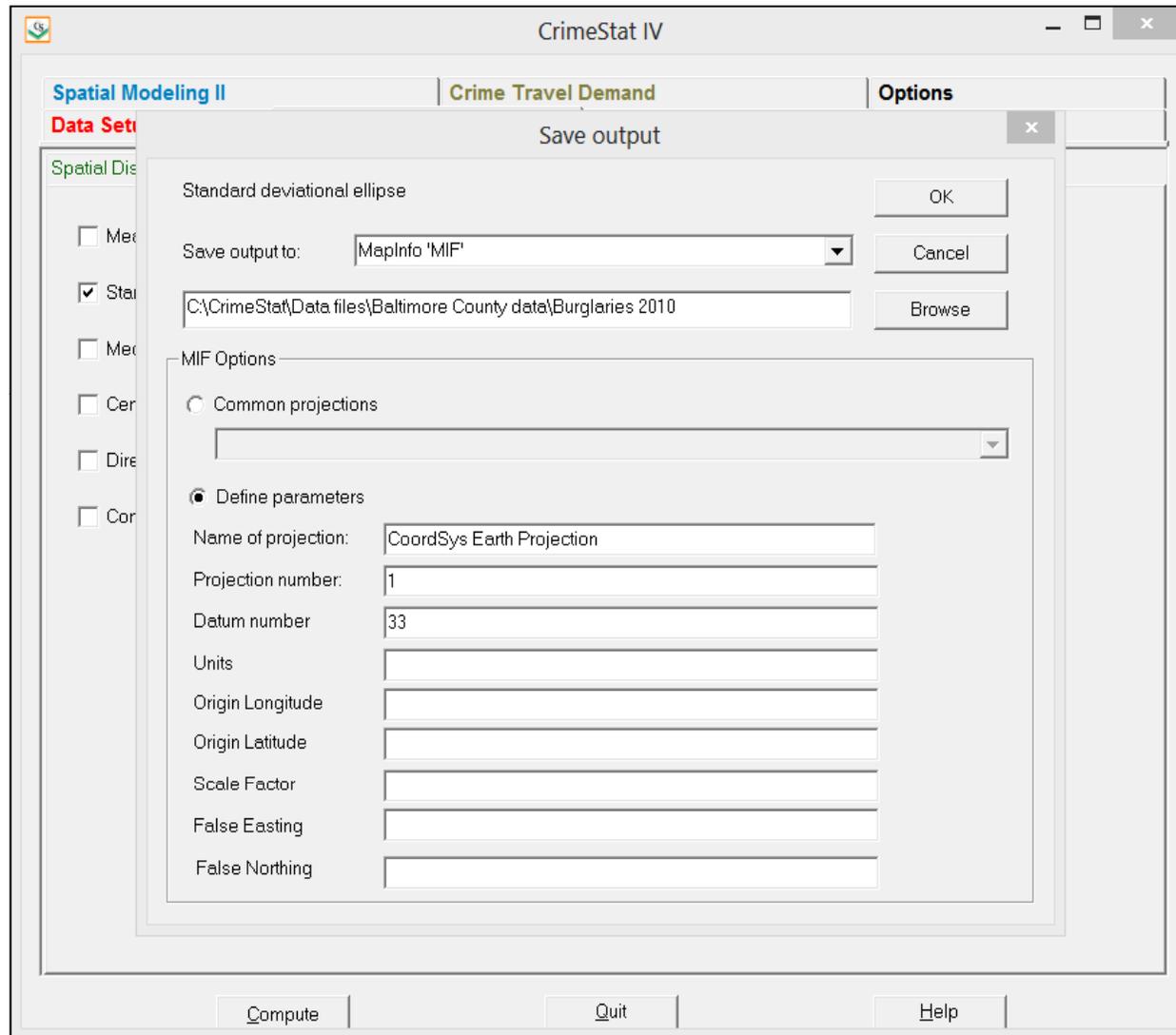


Figure 4.17:  
**MapInfo Output Options**



## Statistical Testing

While the current version of *CrimeStat* does not conduct statistical tests that compare two distributions, it is possible to conduct such tests. Appendix A presents a discussion of the statistical tests that can be used. Instead, the discussion here will focus on using the outputs of the routines without formal testing.

### Decision-making Without Formal Tests

Formal significance testing has the advantage of providing a consistent inference about whether the difference in two distributions is likely or unlikely to be due to chance. Almost all formal tests compare the distribution of a statistic with that of a random distribution. However, police departments frequently have to make decisions based on small samples, in which case the formal tests are less useful than they would with larger samples. Still, the centrographic statistics calculated in *CrimeStat* can be useful and can help a police department make decision even in the absence of formal tests.

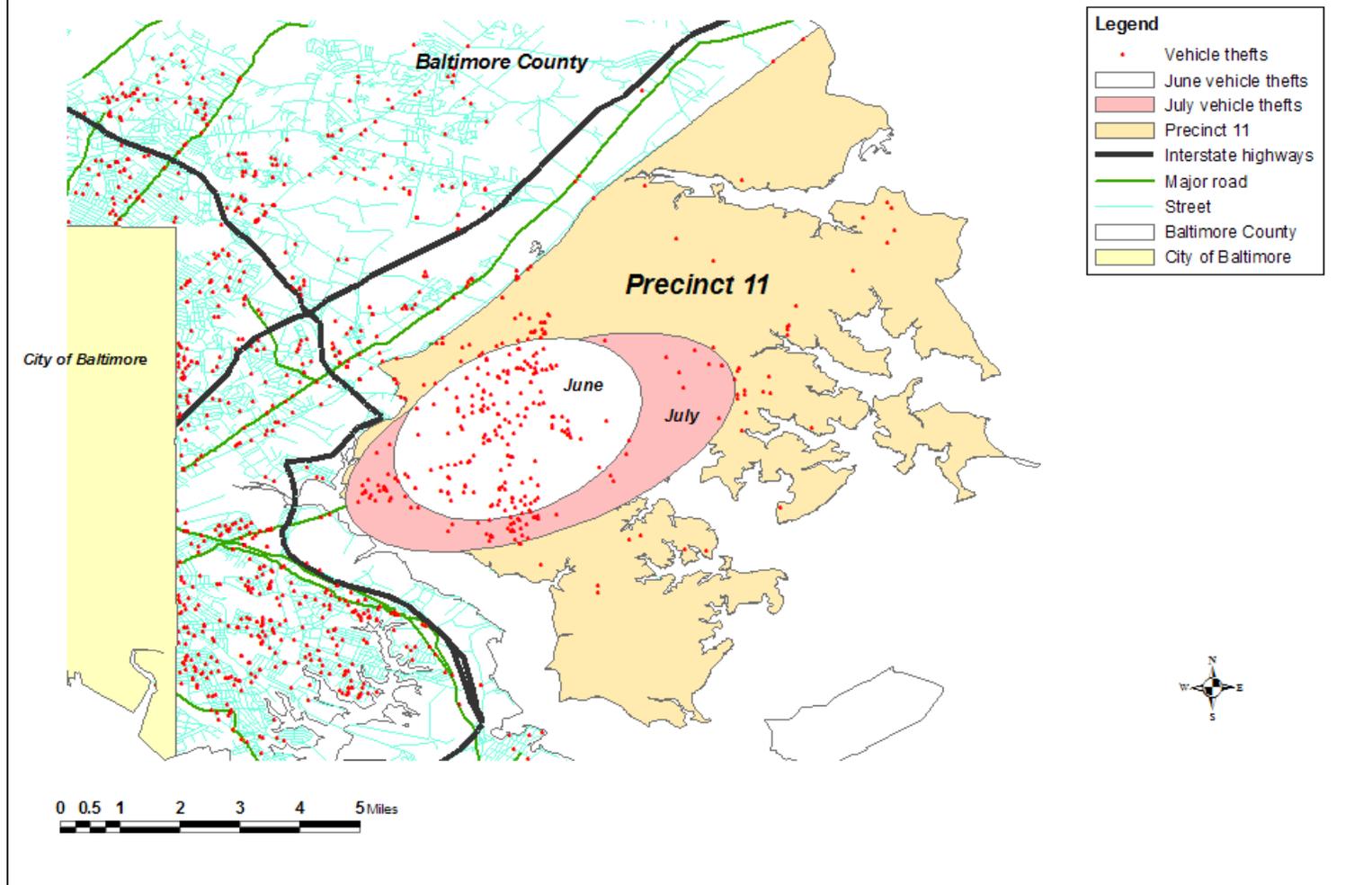
## Examples of Centrographic Statistics

### Example 1: June and July Auto Thefts in Precinct 11

We want to illustrate the use of these statistics to make decisions with two examples. The first is a comparison of crimes in small geographical areas. In most metropolitan areas, most analysts will concentrate on particular sub-areas of the jurisdiction, rather than on the jurisdiction itself. In Baltimore County, for instance, analysis is done both for the jurisdiction as a whole as well as by individual precincts.

Below in Figure 4.18 are the standard deviational ellipses for 1996 auto thefts for June and July in Precinct 11 of Baltimore County. As can be seen, there was a spatial shift that occurred between June and July of that year, the result most probably of increased vacation travel to the Chesapeake Bay. While the comparison is very simple, involving looking at the graphical object created by *CrimeStat*, such a month to month comparison can be useful for police departments because it points to a shift in incident patterns, allowing the police department to reorient their patrol units.

Figure 4.18:  
**Vehicle Theft Change in Precinct 11**  
Standard Deviation Ellipses for June and July 1996



## Example 2: Serial Burglaries in Baltimore City and Baltimore County

The second example illustrates a rash of burglaries that occurred on both sides of the border of Baltimore City and Baltimore County. On one hand there were ten residential burglaries that occurred on the western edge of the City/County border within a short time period of each other and, on the other hand, there were 13 commercial burglaries that occurred in the central part of the metropolitan areas. Both police departments suspected that these two sets were the work of a serial burglar (or group of burglars). What they were not sure about was whether the two sets of burglaries were done by the same individuals or by different individuals.

The number of incidents involved are too small for significance testing; only one of the parameters tested was significant and that could easily be due to chance. However, the police do have to make a guess about the possible perpetrator even with limited information. Let's use *CrimeStat* to try and make a decision about the distributions.

Figure 4.19 illustrates these distributions. The thirteen commercial burglaries are shown as squares while the ten residential burglaries are shown as triangles. Figure 4.20 plots the mean centers of the two distributions. They are close to each other, but not identical. An initial hunch would suggest that the robberies are committed by two perpetrators (or groups of perpetrators), but the mean centers are not different enough to truly confirm this expectation.

Similarly, Figure 4.21 plots the center of minimum distance. Again, there is a difference in the distribution, but it is not great enough to truly rule out the single perpetrator theory. Figure 4.22 plots the raw standard deviations, expressed as a rectangle by *CrimeStat*. The dispersion of incidents overlaps to a sizeable extent and the area defined by the rectangle is approximately the same. In other words, the search area of the perpetrator or perpetrators is approximately the same. This might argue for a single perpetrator, rather than two. Figure 4.23 shows the standard distance deviation of the two sets of incidents. Again, there is sizeable overlap and the search radiuses are approximately the same.

Only with the standard deviational ellipse, however, is there a fundamental difference between the two distributions (figure 4.24). The pattern of commercial robberies is falling along a northeast-southwest orientation while that for residential robberies along a northwest-southeast axis. In other words, when the orientation of the incidents is examined, as defined by the standard deviational ellipse, there are two completely opposite patterns. Unless this difference can be explained by an obvious factor (e.g., the distribution of commercial establishments), it is probable that the two sets of robberies were committed by two different perpetrators (or groups of perpetrators).

Figure 4.19:  
**Profiling Serial Burglars**  
Incident Distribution of Two Serial Burglars

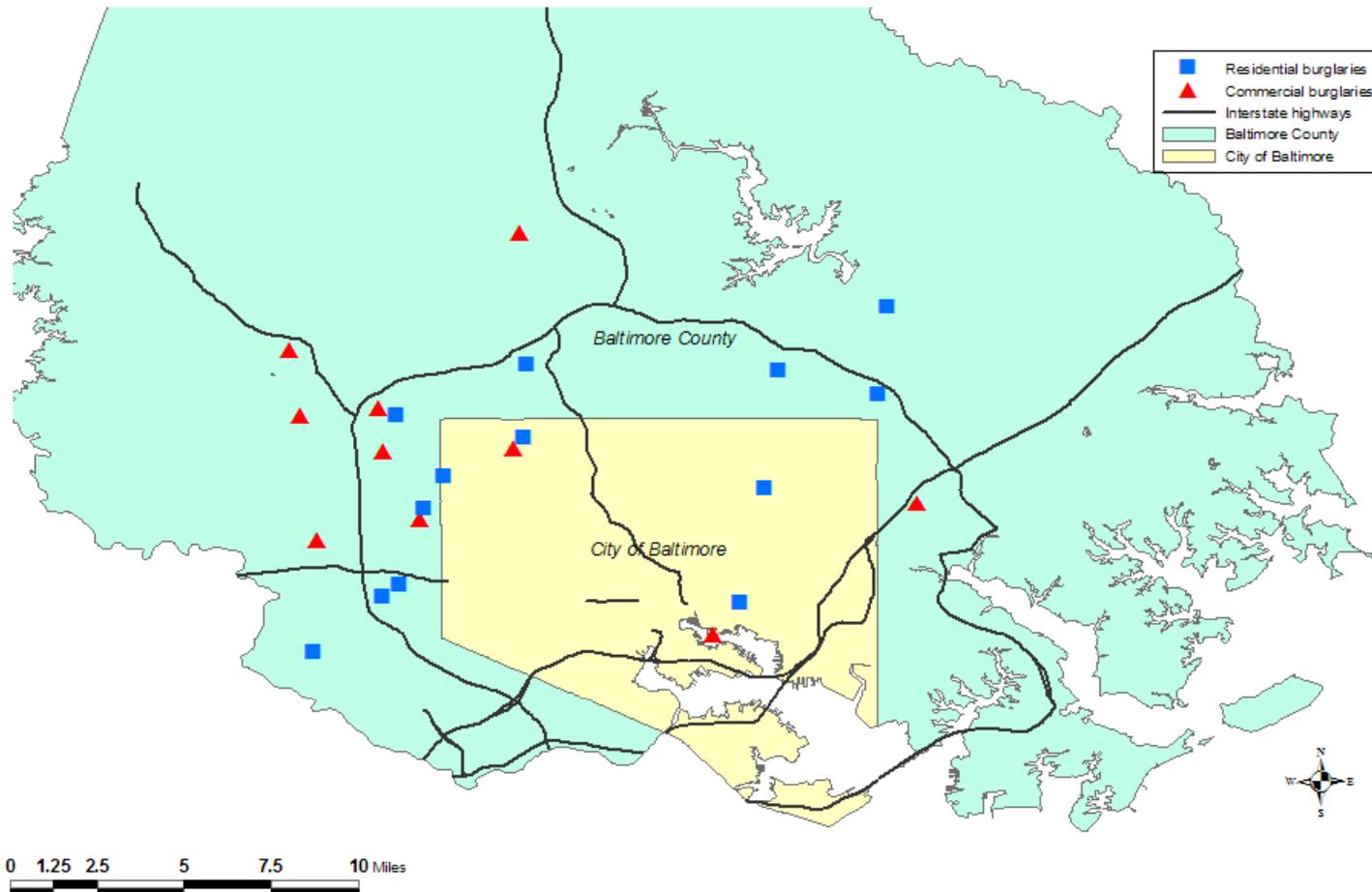


Figure 4.20:  
**Profiling Serial Burglars**  
Mean Center for Two Serial Burglars

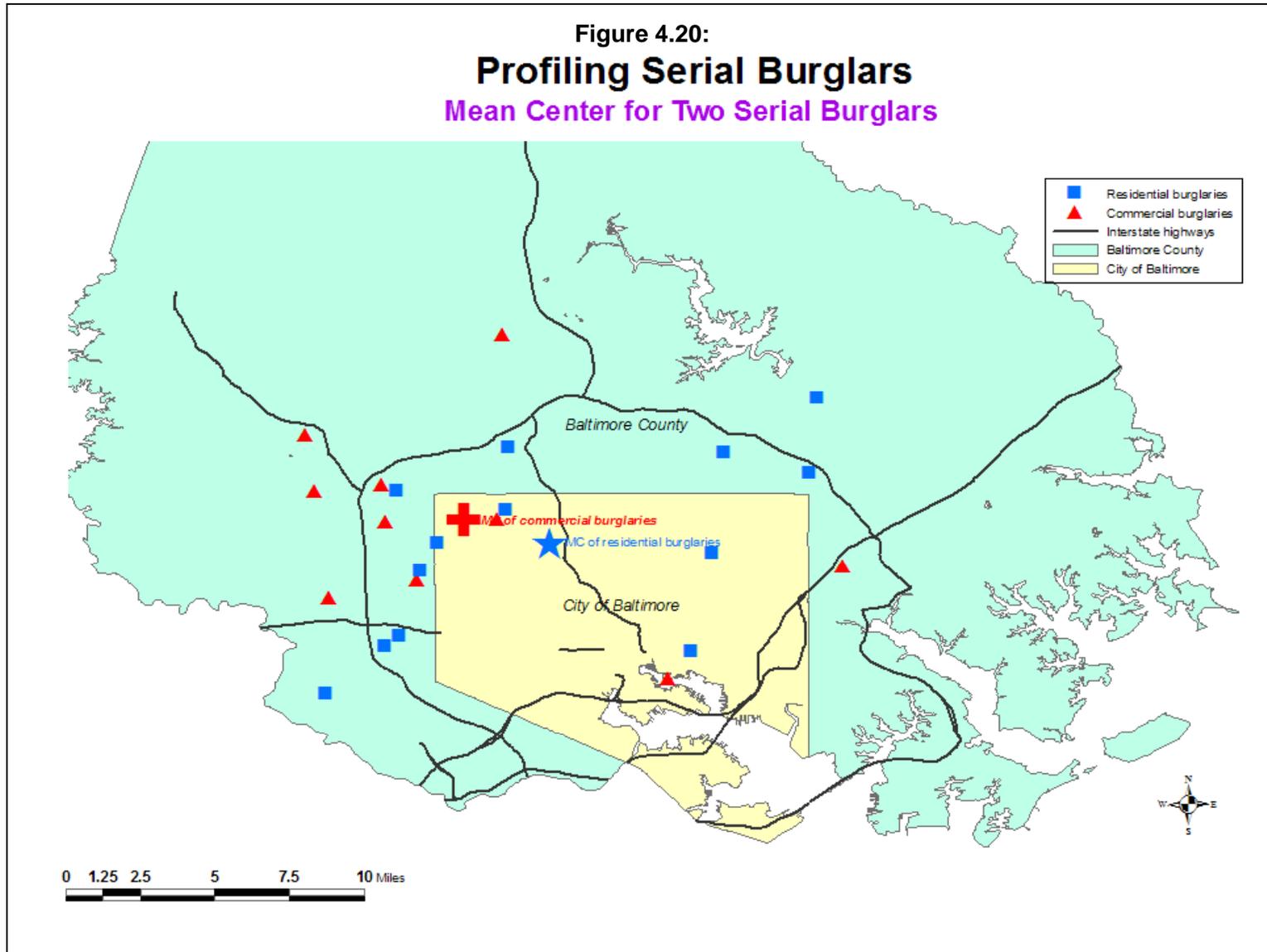


Figure 4.21:  
**Profiling Serial Burglars**  
Center of Minimum Distance for Two Serial Burglars

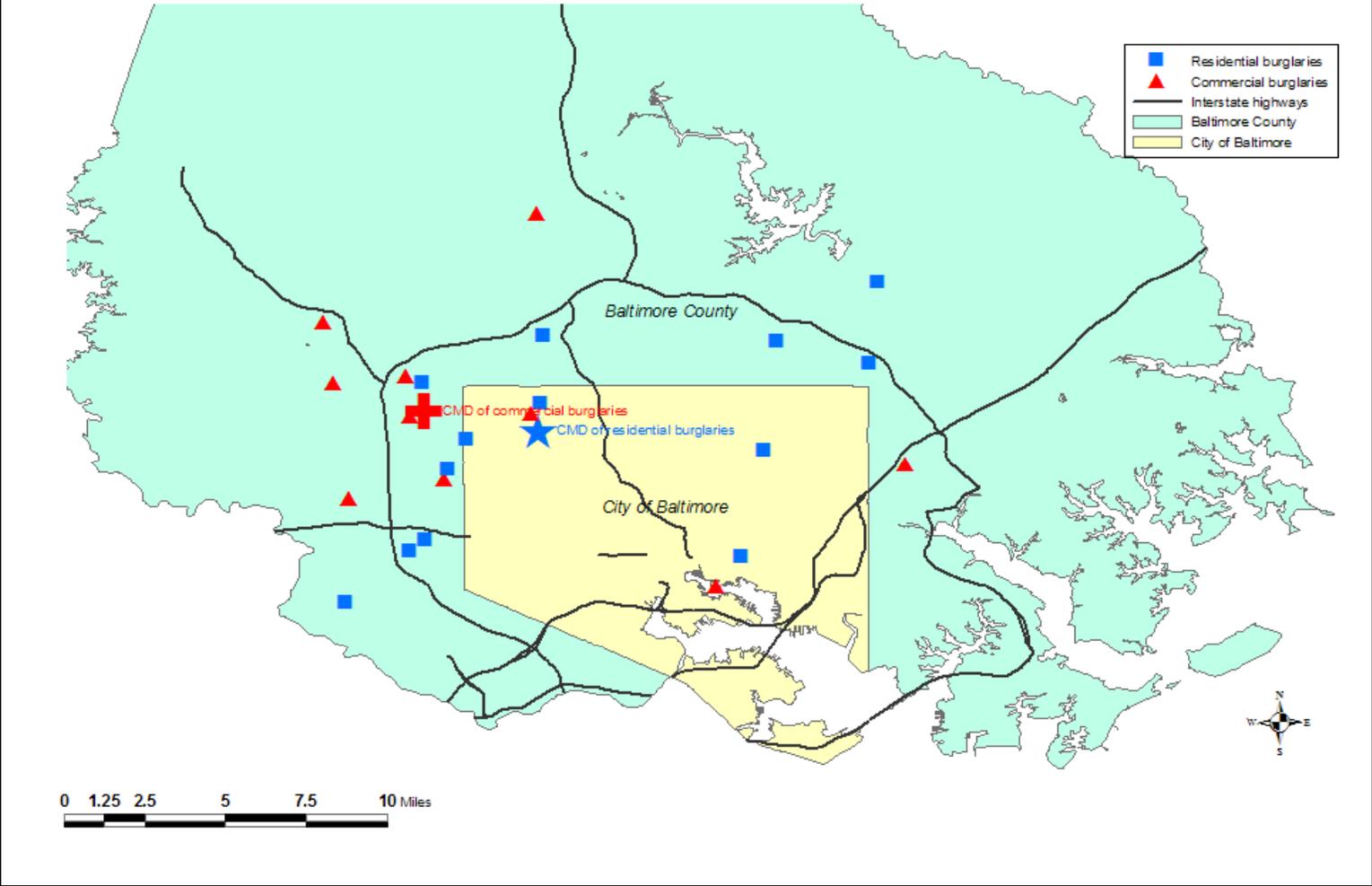
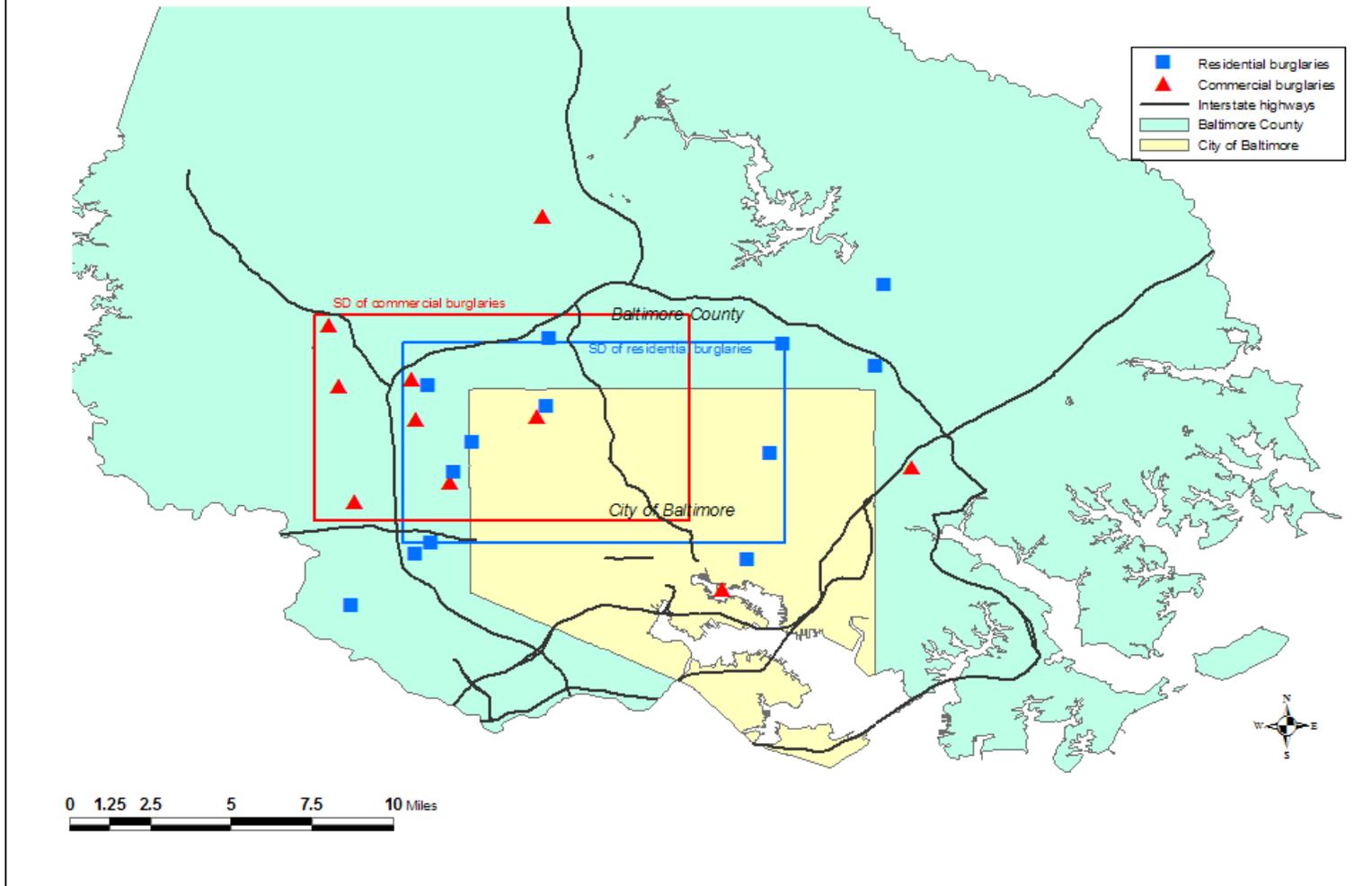


Figure 4.22:  
**Profiling Serial Burglars**  
Standard Deviations of Incidents for Two Serial Burglars



**Figure 4.23:**  
**Profiling Serial Burglars**  
Standard Distance Deviation of Incidents for Two Serial Burglars

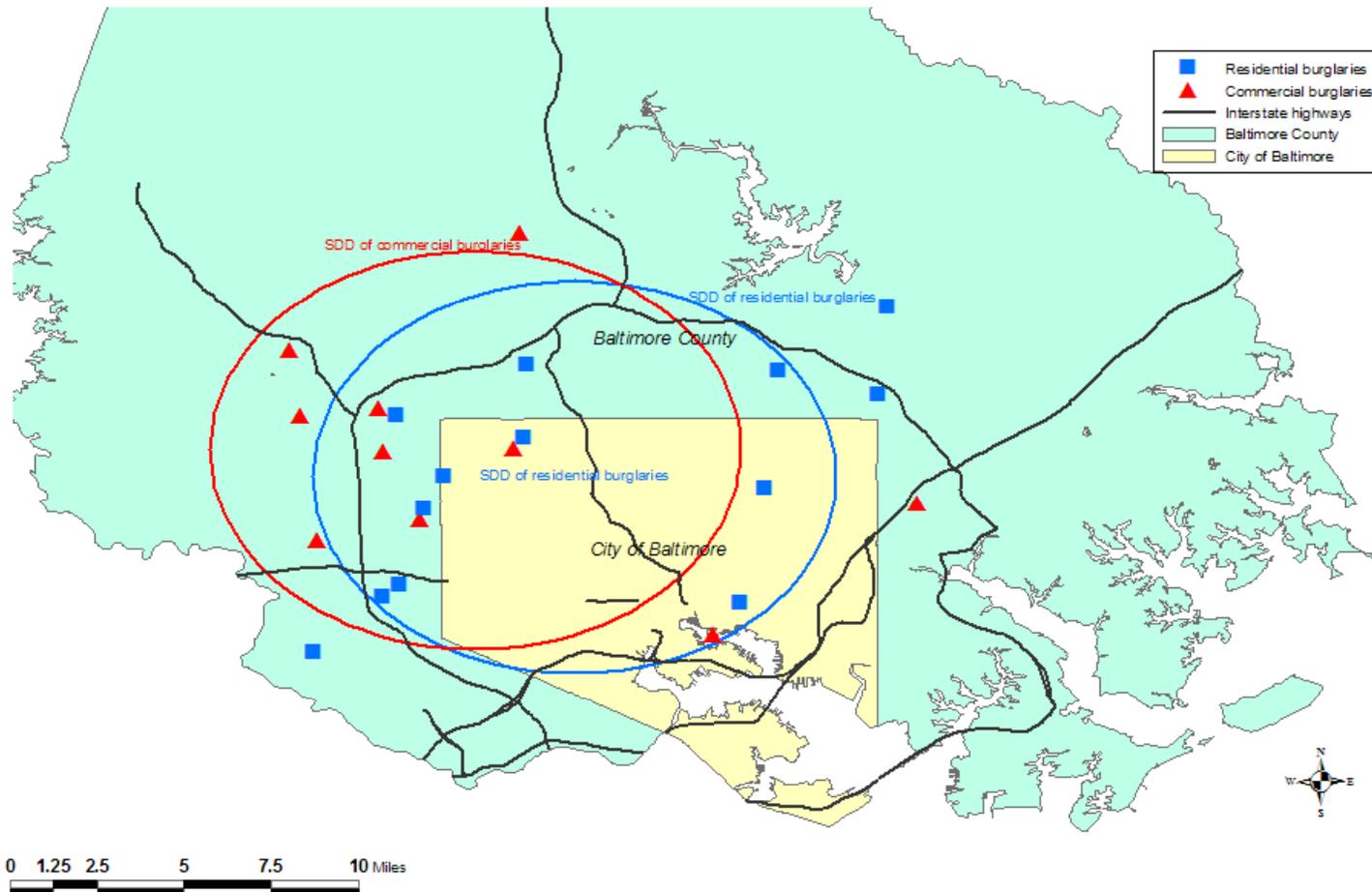
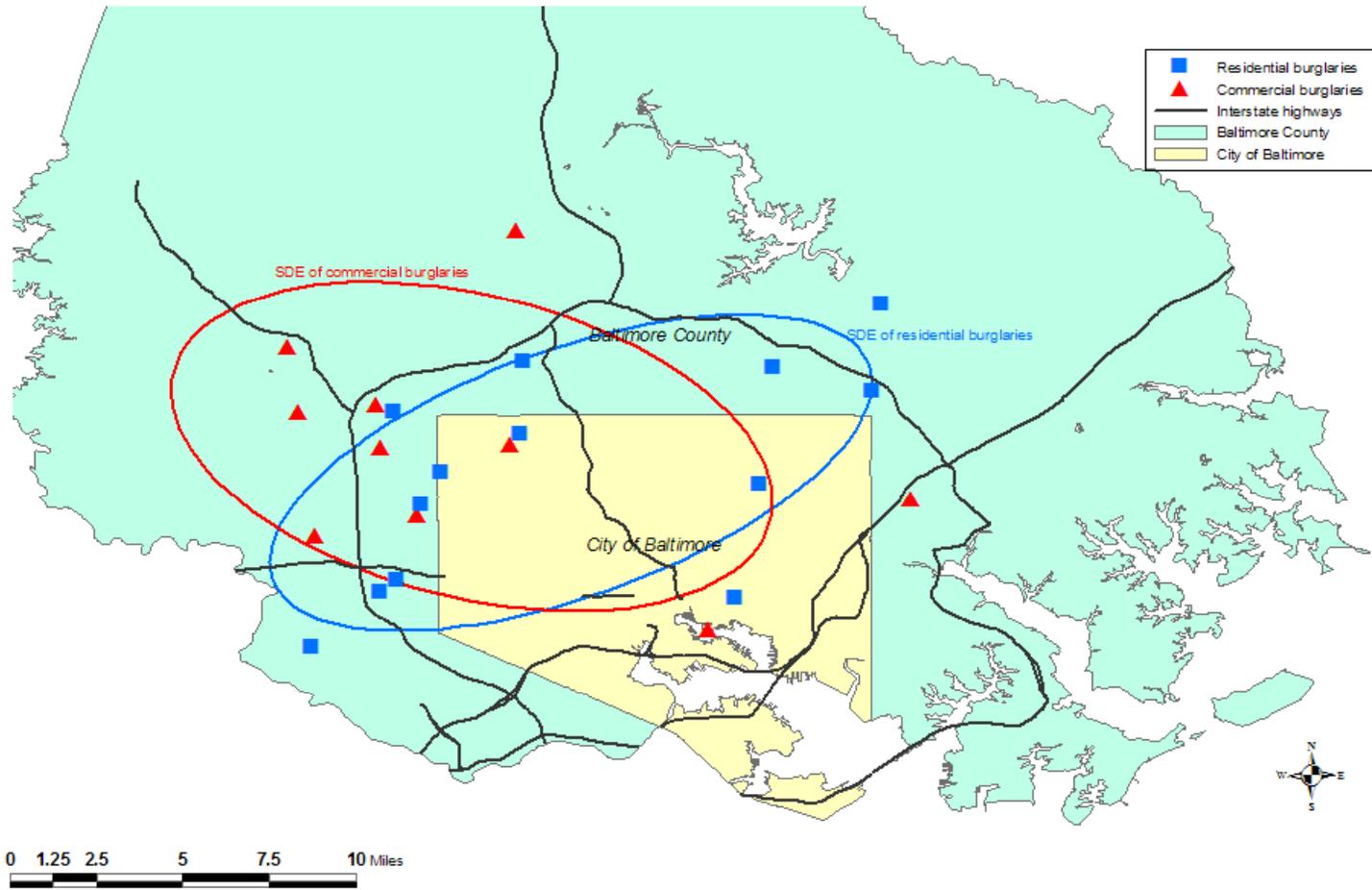


Figure 4.24:  
**Profiling Serial Burglars**  
Standard Deviational Ellipse of Incidents for Two Serial Burglars



## Directional Mean and Variance

Centrographic statistics utilize the coordinates of a point, defined as an X and Y value on either a spherical or projected/Cartesian coordinate system. There is another type of metric that can be used for identifying incident locations, namely a **polar coordinate** system. A *vector* is a line with direction and length. In this system, there is a reference vector (usually  $0^0$  due North) and all locations are defined by angular deviations from this reference vector. By convention, angles are defined as deviations from  $0^0$ , clockwise through  $360^0$ . Note the measurement scale is a circle which returns back on itself (i.e.  $0^0$  is also  $360^0$ ). Point locations can be represented as vectors on a polar coordinate system.

With such a system, ordinary statistics cannot be used. For example, if there are five points which on the northern side of the polar coordinate system and are defined by their angular deviations as  $0^0$ ,  $10^0$ ,  $15^0$ ,  $345^0$ , and  $350^0$  from the reference vector (moving clockwise from due North), the statistical mean will produce an erroneous estimate of  $144^0$ . This vector would be southeast and will lie in an opposite direction from the distribution of points.

Instead, statistics have to be calculated by trigonometric functions. The input for such a system is a set of vectors, defined as angular deviations from the reference vector and a distance vector. Both the angle and the distance vector are defined with respect to an origin. The routine can calculate angles directly or can convert all X and Y coordinates into angles with a bearing from an origin. For reading angles directly, the input is a set of vectors, defined as angular deviations from the reference vector. *CrimeStat* calculates the mean direction and the circular variance of a series of points defined by their angles. On the primary file screen, the user must select Direction (angles) as the coordinate system.

If the angles are to be calculated from X/Y coordinates, the user must define an origin location. On the reference file page, the user can select among three origin points:

1. The lower-left corner of the data set (the minimum X and Y values). This is the default setting.
2. The upper-right corner of the data set (the maximum X and Y values); and
3. A user-defined point.

Users should be careful about choosing a particular location for an origin, either lower-left, upper-right or user-defined. If there is a point at that origin, *CrimeStat* will drop that case since any calculations for a point with zero distance are indeterminate. Users should check that

there is no point at the desired origin. If there is, then the origin should be adjusted slightly so that no point falls at that location (e.g., taking slightly smaller X and Y values for the lower-left corner or slightly larger X and Y values for the upper right corner).

The routine converts all X and Y points into an angular deviation from true North relative to the specified origin and a distance from the origin. The bearing is calculated with different formulae depending on the quadrant that the point falls within.

### **First Quadrant**

With the lower-left corner as the origin, all angles are in the first quadrant. The clockwise angle,  $\theta_i$ , is calculated by:

$$\theta = \arctan \left[ \frac{\text{Abs}(X_i - X_O)}{\text{Abs}(Y_i - Y_O)} \right] \quad (4.24)$$

where  $X_i$  is the X-value of the point,  $Y_i$  is the Y-value of the point,  $X_O$  is the X-value of the origin, and  $Y_O$  is the Y-value of the origin.

The angle,  $\theta_i$ , is in radians and can be converted to polar coordinate degrees using:

$$\theta_{degrees} = \theta_{radians} \frac{180}{\pi} \quad (4.25)$$

### **Third Quadrant**

With the upper-right corner as the origin, all angles are in the third quadrant. The clockwise angle,  $\theta_i$ , is calculated by:

$$\theta_{radians} = \pi + \arctan \left[ \frac{\text{Abs}(X_i - X_O)}{\text{Abs}(Y_i - Y_O)} \right] \quad (4.26)$$

where the angle,  $\theta_i$ , is again in radians. Since there are  $2\pi$  radians in a circle,  $\pi$  radians is  $180^\circ$ . Again, the angle in radians can be converted into degrees with formula 4.25 above.

### **Second and Fourth Quadrants**

When the origin is user-defined, each point must be evaluated as to which quadrant it is in. The second and fourth quadrants define the clockwise angle,  $\theta_i$ , differently:

### ***Second quadrant***

$$\theta_{radians} = 0.5\pi + \arctan \left[ \frac{Abs(Y_i - Y_0)}{Abs(X_i - X_0)} \right] \quad (4.27)$$

### ***Fourth quadrant***

$$\theta_{radians} = 1.5\pi + \arctan \left[ \frac{Abs(Y_i - Y_0)}{Abs(X_i - X_0)} \right] \quad (4.28)$$

Once all X/Y coordinates are converted into angles, the mean angle is calculated.

### **Mean Angle**

With either angular input or conversion from X/Y coordinates, the *Mean Angle* is the resultant of all individual vectors (i.e., points defined by their angles from the reference vector). It is an angle that summarizes the mean direction. Graphically, a *resultant* is the sum of all vectors and can be shown by laying each vector end to end. Statistically, it is defined as

$$Mean\ angle = \bar{\theta}_{radians} = Abs \left\{ \arctan \left[ \frac{\sum d_i \sin \theta_i}{\sum d_i \cos \theta_i} \right] \right\} \quad (4.29)$$

where the summation of sines and cosines is over the total number of points,  $i$ , defined by their angles,  $\theta_i$ . Each angle,  $\theta_i$ , can be weighted by the length of the vector,  $d_i$ . In an unweighted angle,  $d_i$  is assumed to be of equal length, 1. The absolute value of the ratio of the sum of the weighted sines to the sum of the weighted cosines is taken. All angles are in radians. In determining the mean angle, the quadrant of the resultant must be identified:

1. If  $\Sigma(\sin\theta_i) > 0$  and  $\Sigma(\cos\theta_i) > 0$ , then  $\bar{\theta}$  can be used directly as the mean angle.
2. If  $\Sigma(\sin\theta_i) > 0$  and  $\Sigma(\cos\theta_i) < 0$ , then  $\bar{\theta}$  is  $\pi/2 + \theta$ .
3. If  $\Sigma(\sin\theta_i) < 0$  and  $\Sigma(\cos\theta_i) < 0$ , then  $\bar{\theta}$  is  $\pi + \theta$ .
4. If  $\Sigma(\sin\theta_i) < 0$  and  $\Sigma(\cos\theta_i) > 0$ , then  $\bar{\theta}$  is  $1.5\pi + \theta$ .

Formulas 4.26, 4.27, 4.28 and 4.29 above are then used to convert the directional mean back to an X/Y coordinate, depending on which quadrant it falls within.

## Circular Variance

The dispersion (or variance) of the angles are also defined by trigonometric functions. The unstandardized variance,  $R$ , is sometimes called the *sample resultant length* since it is the resultant of all vectors (angles):

$$R = \sqrt{[\sum(d_i \sin\theta_i)^2] + [\sum(d_i \cos\theta_i)^2]} \quad (4.30)$$

where  $d_i$  is the length of vector,  $i$ , with an angle (bearing) for the vector of  $\theta_i$ . For the unweighted sample resultant,  $d_i$  is 1.

Because  $R$  increases with sample size, it is standardized by dividing by  $N$  to produce a *mean resultant length*:

$$\bar{R} = \frac{R}{N} \quad (4.31)$$

where  $N$  is the number points (sample size).

Finally, the average distance from the origin,  $D$ , is calculated and the *circular variance* is calculated by:

$$\text{Circular variance} = \frac{1}{D} \left( D - \frac{R}{N} \right) = \frac{D - \bar{R}}{D} = 1 - \frac{\bar{R}}{D} \quad (4.32)$$

This is the standardized variance which varies from 0 (no variability) to 1 (maximum variability). The details of the derivations can be found in Burt and Barber (1996) and Gaile and Barber (1980).

## Mean Distance

The mean distance,  $\bar{d}$ , is calculated directly from the X and Y coordinates. It is identified in relation to the defined origin.

## Directional Mean

The directional mean is calculated as the intersection of the mean angle and the mean distance. It is not a unique position since distance and angularity are independent dimensions. Thus, the directional mean calculated using the minimum X and minimum Y location as the reference origin (the 'lower left corner') will yield a different location from the directional mean

calculated using the maximum X and maximum Y location as the origin (the 'upper right corner'). There is a weighted and unweighted directional mean. Though *CrimeStat* calculates the location, users should be aware of the non-uniqueness of the location. The unweighted directional mean can be output with a 'Dm' prefix. The weighted directional mean is not output.

### **Triangulated Mean**

The triangulated mean is defined as the intersection of the two vectors, one from the lower-left corner of the study area (the minimum X and Y values) and the other from the upper-right corner of the study area (the maximum X and Y values). It is calculated by estimating mean angles from each origin (lower left and upper right corners), translating these into equations, and finding the point at which these equations intersect (by setting the two functions equal to each other).

### **Directional Mean Output**

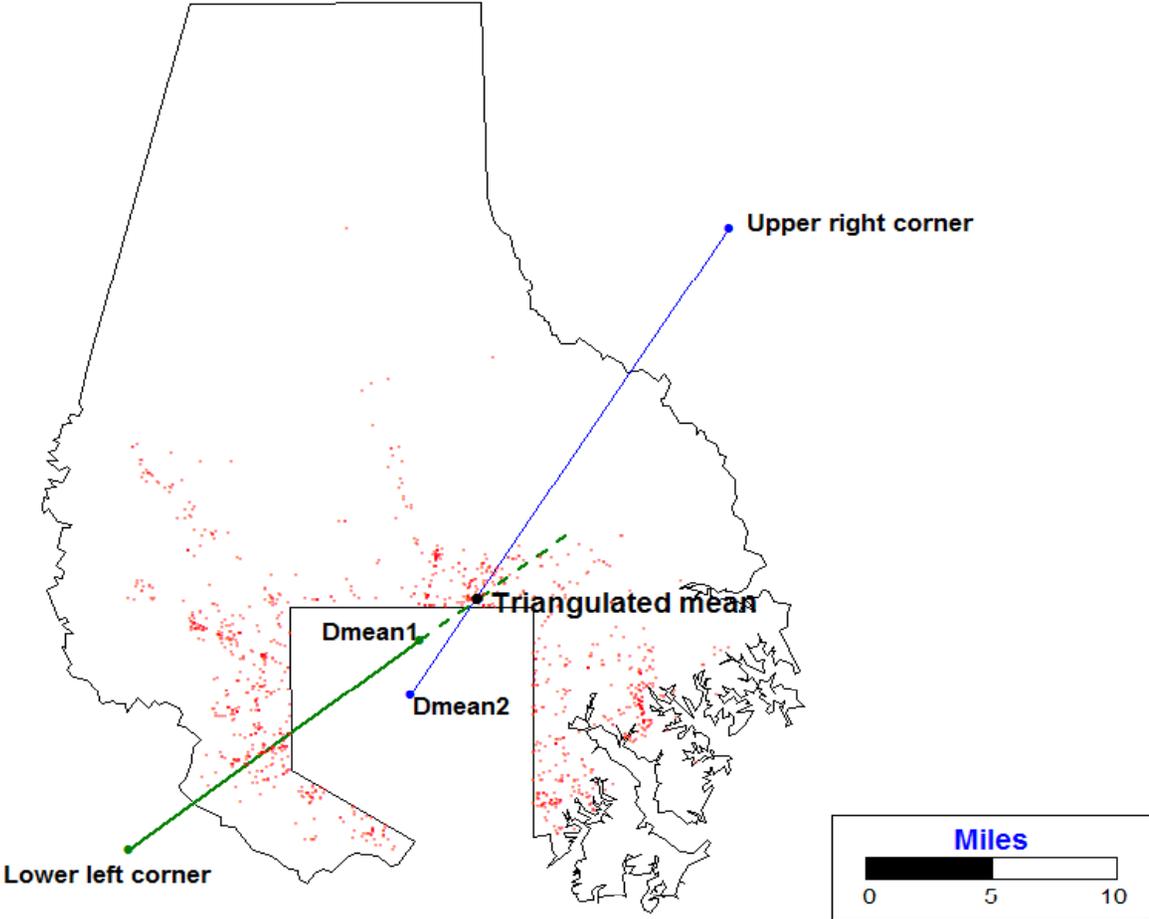
The directional mean routine outputs nine statistics:

1. The sample size;
2. The unweighted mean angle;
3. The weighted mean angle;
4. The unweighted circular variance;
5. The weighted circular variance;
6. The mean distance;
7. The intersection of the mean angle and the mean distance;
8. The X and Y coordinates for the triangulated mean; and
9. The X and Y coordinates for the weighted triangulated mean.

The directional mean and triangulated mean can be saved as *ArcGIS* 'shp', *MapInfo* 'mif', *Google Earth* 'kml', or various Ascii files. The unweighted directional mean - the intersection of the mean angle and the mean distance, is output with the prefix 'Dm' while the unweighted triangulated mean location is output with a 'Tm' prefix. The weighted triangulated mean is output with a 'TmWt' prefix. See the example below.

Figure 4.25 shows the unweighted triangular mean for 1996 Baltimore County robberies and compares it to the two directional means calculated using the lower-left corner (Dmean1) and the upper-right corner (Dmean2) respectively as origins. As can be seen, the two directional means fall at different locations. Lines have been drawn from each origin point to their

Figure 4.25:  
**Triangulated Mean for Baltimore County Robberies**  
Defined by the Intersection of Two Mean Angles



respective directional means and are extended until they intersect. As seen, the triangulated mean falls at the location where the two vectors (i.e., mean angles) intersect.

Because the triangulated mean is calculated with vector geometry, it will not necessarily capture the central tendency of a distribution. Asymmetrical distributions can cause it to be placed in peripheral locations. On the other hand, if the distribution is relatively balanced in each direction, it can capture the center of orientation perhaps better than other means, as figure 4.25 shows. Appendix A includes a discussion of how to formally test the mean direction between two different distributions.

## Convex Hull

The convex hull is a boundary drawn around the distribution of points. It is a relatively simple concept, at least on the surface. Intuitively, it represents a polygon that circumscribes all the points in the distribution such that no point lies outside of the polygon.

The complexity comes because there are different ways to define a convex hull. The most basic algorithm is the *Graham scan* (Graham, 1972). Starting with one point known to be on the convex hull, typically the point with the lowest X coordinate, the algorithm sorts the remaining points in angular order around this in a counterclockwise manner. If the angle formed by the next point and the last edge is less than 180 degrees, then that point is added to the hull. If the angle is greater than 180 degrees, then the chain of nodes starting from the last edge must be deleted. The routine proceeds until the hull closes back on itself (de Berg, van Kreveld, Overmans, and Schwarzkopf, 2000).

Many alternative algorithms have been proposed. Among these are the 'gift wrap' (Chand and Kapur, 1970; Skiena, 1997), the Quick Hull, the "Divide and conquer" (Preparata and Hong, 1977), and the incremental (Kallay, 1984) algorithms. Even more complexity has been introduced by the mathematics of fractals where an almost infinite number of borders could be defined (Lam and De Cola, 1993). In most implementations, though, a simplified algorithm is used to produce the convex hull.

*CrimeStat* implements the 'gift wrap' algorithm. Starting with the point with the lowest Y coordinate, A, it searches for another point, B, such that all other points lie to the left of the line AB. It then finds another point, C, such that all remaining points lie to the left of the line BC. It continues in this way until it reaches the original point A again. It is like 'wrapping a gift' around the outside of the points.

The routine outputs three statistics:

1. The sample size;
2. The number of points in the convex hull
3. The X and Y coordinates for each of the points in the convex hull

The convex hull can be saved as *ArcGIS* 'shp', *MapInfo* 'mif', *Google Earth* 'kml', or various Ascii files with a 'Chull' prefix.

Figure 4.26 shows the convex hull of Baltimore County robberies for 1996. As seen, the hull occupies a relatively smaller part of Baltimore County. Figure 4.27, on the other hand shows the convex hull of 1996 Baltimore County burglaries. As seen, the convex hull of the burglaries cover a much larger area than for the robberies.

### **Uses and Limitations of the Convex Hull**

A convex hull can be useful for displaying the geographical extent of a distribution. Simple comparisons, such as in Figures 4.26 and 4.27, can show whether one distribution has a greater extent than another. Further, as we shall see, a convex hull can be useful for describing the geographical spread of a crime hot spot, essentially indicating where the crimes are distributed.

On the other hand, a convex hull is vulnerable to extreme values. If one incident is isolated, the hull will of necessity be large. The mean center, too, is influenced by extreme values but not to the same extent since it averages the location of all points. The convex hull, on the other hand, is defined by the most extreme points. A comparison of different crime types or the same crime type for different years using the convex hull may only show the variability of the extreme values, rather than any central property of the distribution. Therefore, caution must be used in interpreting the meaning of a hull.

Figure 4.26:  
**Convex Hull of Baltimore County Robberies: 1996**

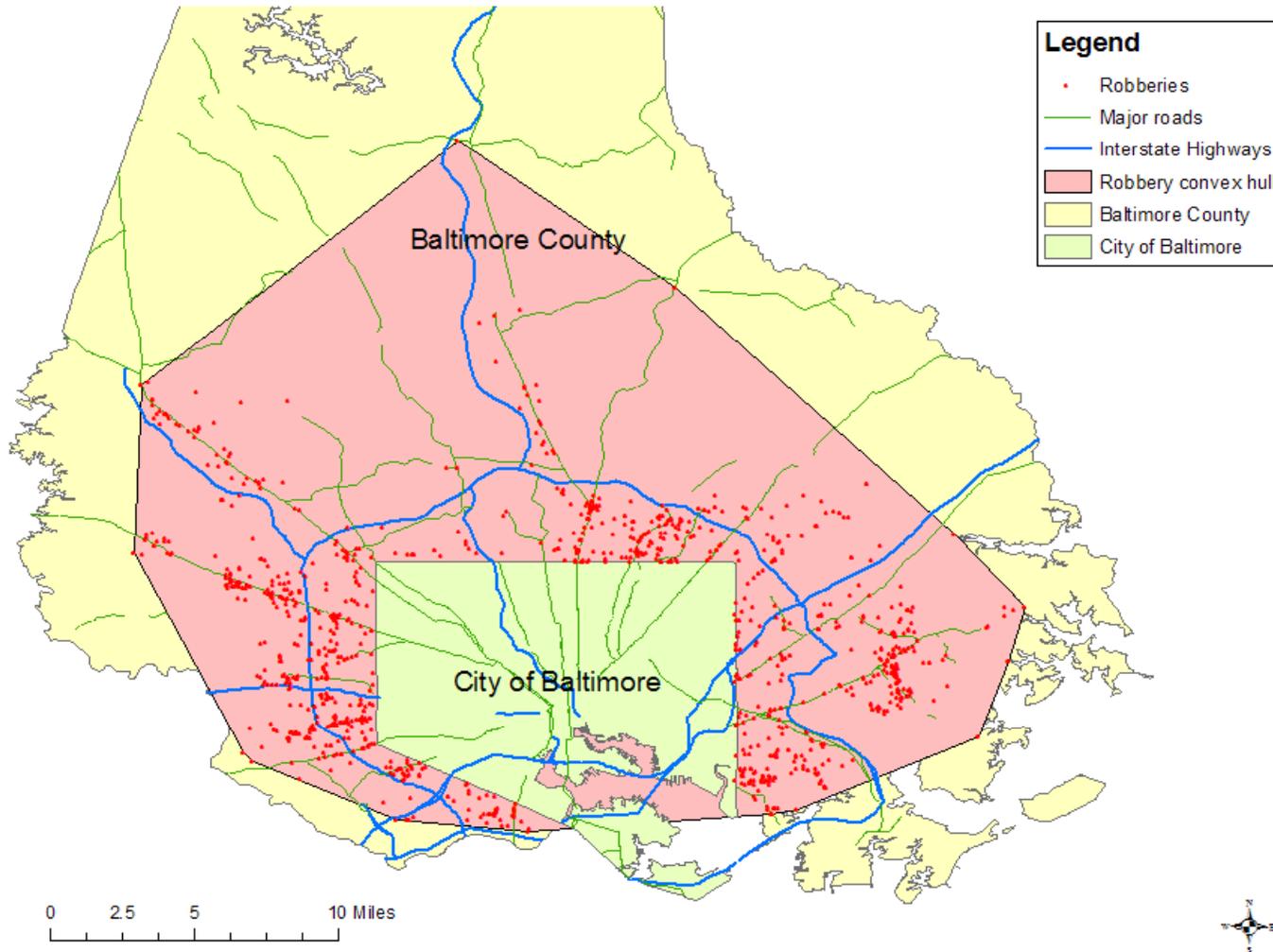
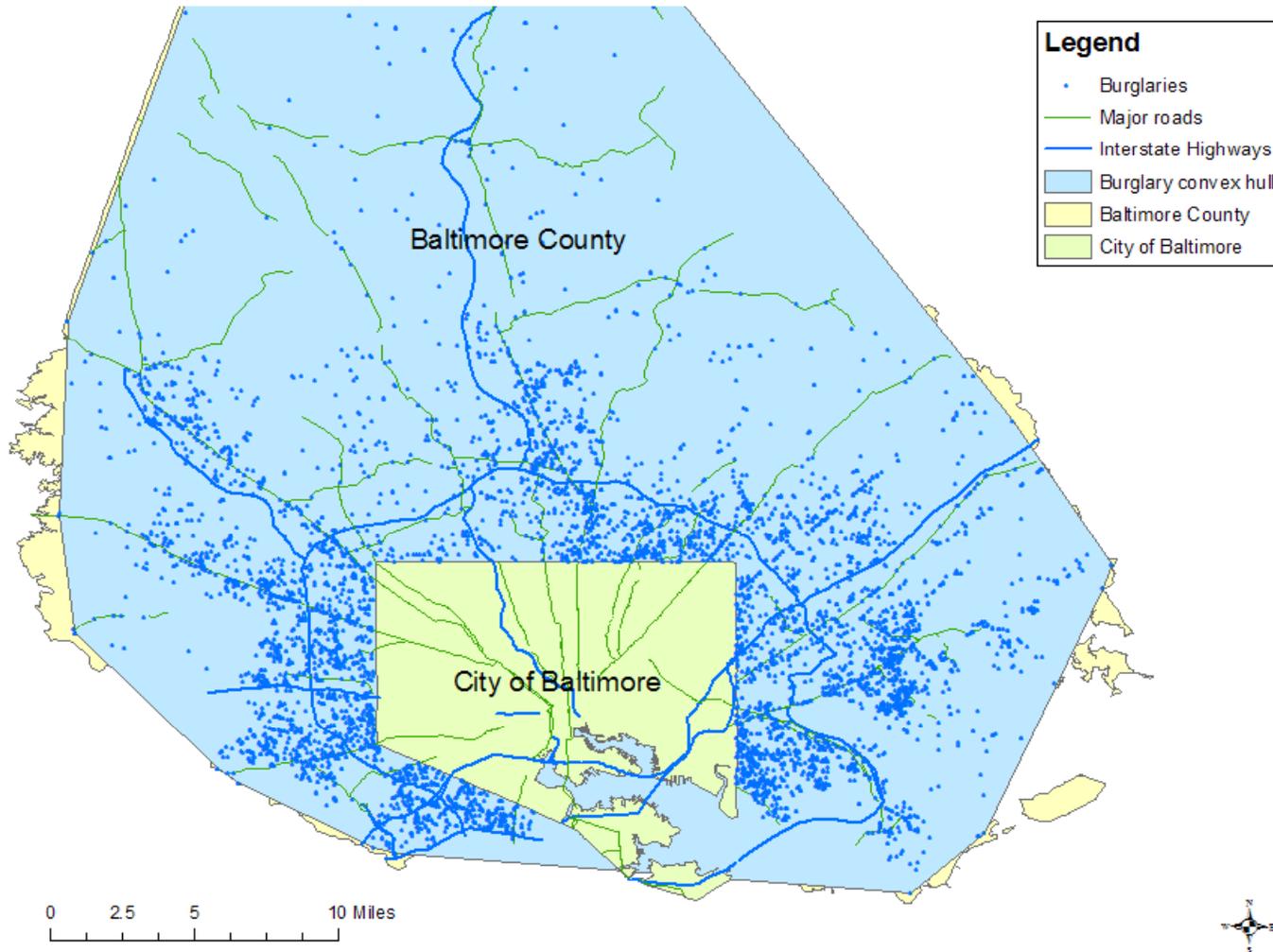


Figure 4.27:  
**Convex Hull of Baltimore County Burglaries: 1996**



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## Endnotes

i. *CrimeStat's* implementation of the Kuhn and Kuenne algorithm is as follows (from Burt and Barber, 1996, 112-113):

1. Let  $t$  be the number of the iteration. For the first iteration only (i.e.,  $t=1$ ) the weighted mean center is taken as the initial estimate of the median location,  $X_t$  and  $Y_t$ .
2. Calculate the distance from each point,  $i$ , to the current estimate of the median location,  $d_{ict}$ , where  $i$  is a single point and  $ct$  is the current estimate of the median location during iteration  $t$ .

- a. If the coordinates are spherical, then Great Circle distances are used.
- b. If the coordinates are projected, then Euclidean distances are used.

3. Weight each case by a weight,  $W_i$ , and calculate:

$$K_{it} = W_i e^{-d_{ict}}$$

where  $e$  is the base of the natural logarithm(2.7183..).

- a. If no weights are defined in the primary file,  $W_i$  is assumed to be 1.
- b. If weights are defined in the primary file,  $W_i$  takes their values.

Note that as the distance,  $d_{ict}$ , approaches 0, then  $e^{-d_{ict}}$  becomes 1.

4. Calculate a new estimate of the center of minimum distance from:

$$X^{t+1} = \frac{\sum K_{it} X_i}{\sum K_{it}} \quad \text{for } i=1 \dots n$$

$$Y^{t+1} = \frac{\sum K_{it} Y_i}{\sum K_{it}} \quad \text{for } i=1 \dots n$$

where  $X_i$  and  $Y_i$  are the coordinates of point  $i$  (either lat/lon for spherical or feet or meters for projected).

## Endnotes (continued)

5. Check to see how much change has occurred since the last iteration

$$\text{Abs}|X^{t+1} - X^t| \leq 0.000001$$

$$\text{Abs}|Y^{t+1} - Y^t| \leq 0.000001$$

- a. If either the X or Y coordinates have changed by greater than 0.000001 between iterations, substitute  $X^{t+1}$  for  $X^t$  and  $Y^{t+1}$  for  $Y^t$  and repeat steps B through D.
- b. If *both* the change in X and the change in Y is less than or equal to 0.000001, then the estimated  $X_t$  and  $Y_t$  coordinates are taken as the center of median distance.

- ii. Formulas for the new axes provided by Ebdon (1988) and Cromley (1992) yield standard deviational ellipses that are too small, for two different reasons. First, they produce transformed axes that are too small. If the distribution of points is random and even in all directions, ideally the standard deviational ellipse should be equal to the standard distance deviation, since  $S_x = S_y$ . The formula used here has this property. Since the formula for the standard distance deviation is (equation 4.8):

$$SDD = \sqrt{\left[ \frac{\sum(X_i - \bar{X})^2 + (Y_i - \bar{Y})^2}{N-2} \right]}$$

If  $S_x = S_y$ , then  $\sum(X_i - X)^2 = \sum(Y_i - Y)^2$ , therefore:

$$SDD = \sqrt{2 \frac{\sum(X_i - \bar{X})^2}{N-2}}$$

Similarly, the formulas for the transformed axes are (4.9, 4.10):

$$S_X = \sqrt{2 \frac{\sum[(X_i - \bar{X})\cos\theta - (Y_i - \bar{Y})\sin\theta]^2}{N-2}}$$

$$S_Y = \sqrt{2 \frac{\sum[(X_i - \bar{X})\sin\theta + (Y_i - \bar{Y})\cos\theta]^2}{N-2}}$$

## Endnotes (continued)

However, if  $S_x = S_y$ , then  $\theta = 0$ ,  $\cos \theta = 1$ ,  $\sin \theta = 0$  and, therefore:

$$s_X = s_Y = \sqrt{2 \frac{\sum (X_i - \bar{X})^2}{N-2}}$$

which is the same as for the standard distance deviation (SDD) under the same conditions. The formulas used by Ebdon (1988) and Cromley (1992) produce axes which are  $\sqrt{2}$  times too small.

The second problem with the Ebdon and Cromley formulas is that they do not correct for degrees of freedom and, hence, produce too small a standard deviational ellipse. Since there are two constants in each equation,  $\bar{X}$  and  $\bar{Y}$ , then there are only  $N-2$  degrees of freedom. The cumulative effect of using transformed axes that are too small and not correcting for degrees of freedom yields a much smaller ellipse than that used here.

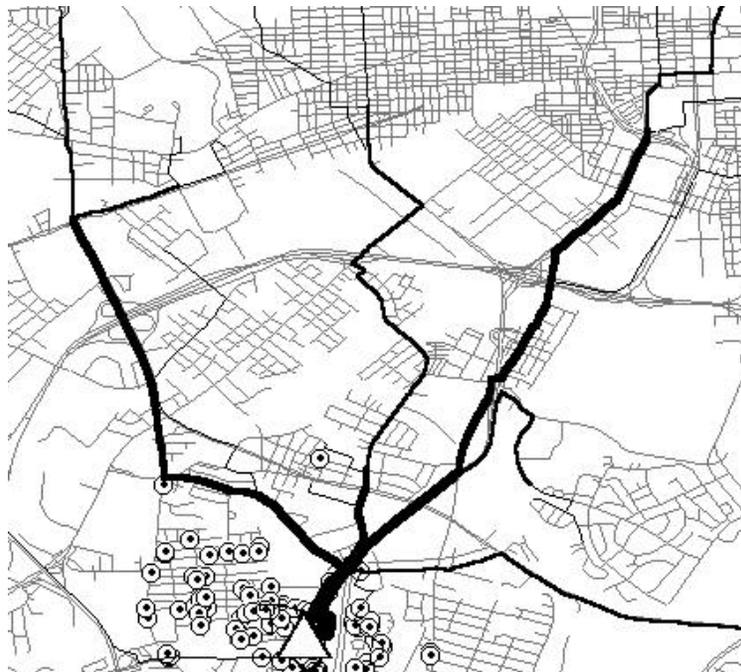
## **Attachments**

## Using Spatial Measures of Central Tendency with Network Analyst to Identify Routes Used by Motor Vehicle Thieves

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Motor vehicle thefts have been steadily declining countywide over the last 5 years, but one police precinct in southwest Baltimore County was experiencing significant increases over several months. Cases were concentrated in several communities, but directed deployment and saturated patrols had minimal impact. In addition to increasing patrols in target communities, the precinct commander was interested in deploying police on roads possibly used by motor vehicle thieves. Police analysts had addresses for theft and recovery locations; it was a matter of using the existing highway network to connect the two locations.

To avoid analyzing dozens of paired locations, analysts decided to set up a database using one location representing the origin of motor vehicle thefts for a particular community. The origin was computed using *CrimeStat's* median center for motor vehicle theft locations reported for a particular community. The median center is the position of minimum average travel and is less affected by extreme locations compared to the arithmetic mean center. The database consisted of the median center paired with a recovery location. Using Network Analyst, a least-effort route was computed for cases reported by community. A count was assigned to each link along a roadway identified by Network Analyst. Analysts used the count to thematically weight links in ArcView. The precinct commander deployed resources along these routes with orders to stop suspicious vehicles. This operation resulted in 27 arrests, and a reduction in motor vehicle thefts.

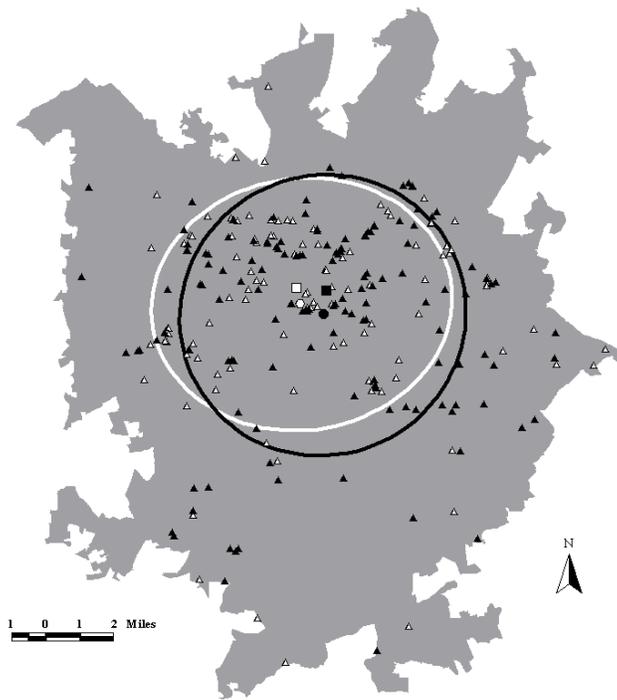


# Centrographic Analysis *Man With A Gun* Calls For Service Charlotte, N.C., 1989

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Hurricane Hugo arrived on Friday, September 22, 1989 in Charlotte, North Carolina. That weekend experienced the highest counts of *Man With A Gun* calls for service for the year. The locations of the calls during the Hugo Weekend are compared with the following New Year's Eve weekend.

*CrimeStat* was used to compare the two weekends. Compared to the New Year's Eve weekend: 1) Hugo's mean and median centers are more easterly; 2) Hugo's ellipse is larger and more circular; and 3) Hugo's ellipse shifts more to the east and southeast. The abrupt spatial change of *Man With A Gun* calls during a natural disaster might indicate more instances of defensive gun use for protection of property.



	Call Location	Mean Center	Median Center	Standard Deviation Ellipse
Hurricane Hugo Weekend September 22-24, 1989: N=146				
New Year's Eve Weekend December 29-31, 1989: N=137				